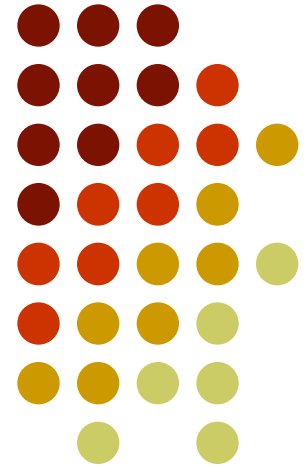


Alpine Verification Meeting 2013, FBK, Trento, Italy.

Optimization Techniques For Craig Interpolant Compaction In Unbounded Model Checking



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Before starting..

- Talk in part based on a paper we presented at DATE 2013 Conference:

Gianpiero Cabodi, C. Loiacono, D. Vendraminetto.
Optimization techniques for Craig interpolant
compaction in unbounded model checking.
DATE 2013: 1417-1422

Outline



- Motivations & background
 - *Hardware designs verification*
 - Craig Interpolants in MC
 - ITP size compaction & scalability
- Contributions
 - Redundancy removal and reduction of
 - UNSAT proofs
 - Craig interpolants
 - Heuristic procedure for scalable ITP compaction
- Experimental results & Conclusions



Motivations

- Can ITPs compete with IC3 ?

IC3	ITP
2-level (AND-OR) characteristic functions	Multiple level circuits
Single instance of TR	TR unrollings

- Main limitations of ITP
 - BMC-based model (vs. cube/clause-based reachability)
 - ITPs are highly redundant



Motivations

- Can ITPs compete with IC3 ?

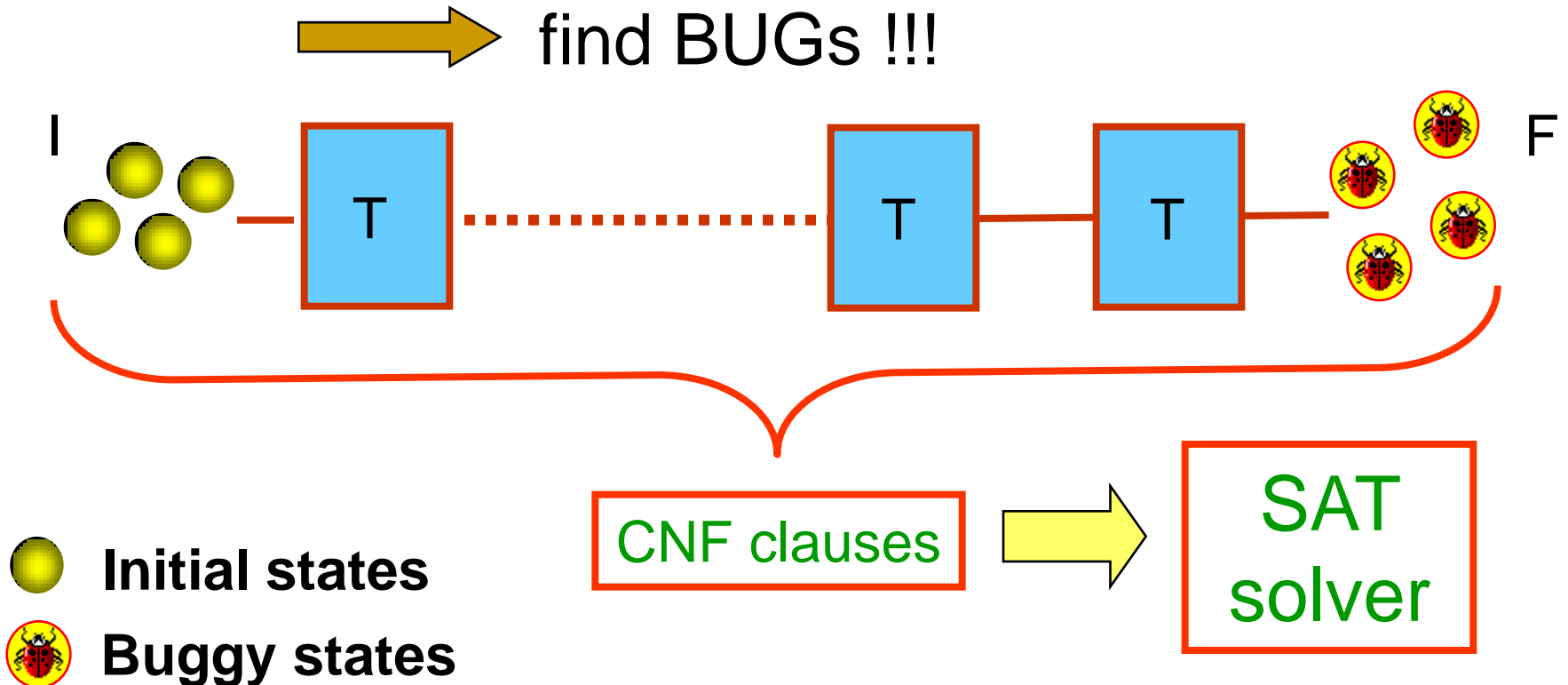
IC3	ITP
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Bounded Model Checking

- Trading off completeness for productivity





Interpolation [Craig'57]

- Given $A \wedge B = 0$
- $A' = \text{interpolant}(A, B)$
 - $A \Rightarrow A'$
 - $A' \wedge B = 0$
 - A' refers only to common variables of A, B
- Interpolants from proofs
 - Given a resolution refutation of $A \wedge B$
 - A' is derived in linear time and space [Pudlak, Krajicek'97]

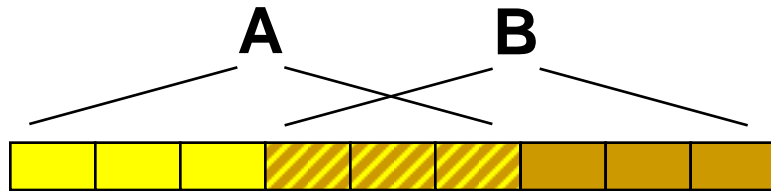


Interpolation [McMillan'03]

- Interpolant as over-approx. image operator
 - Over-approximation
 - Variable quantification
- Works whenever a representation of *backward reachable* space is given
 - A: From $\wedge T$ (FWD)
 - B: paths to failure states (BWD)
 - A': over-approx image
- Approx image is called *adequate* w.r.t. B



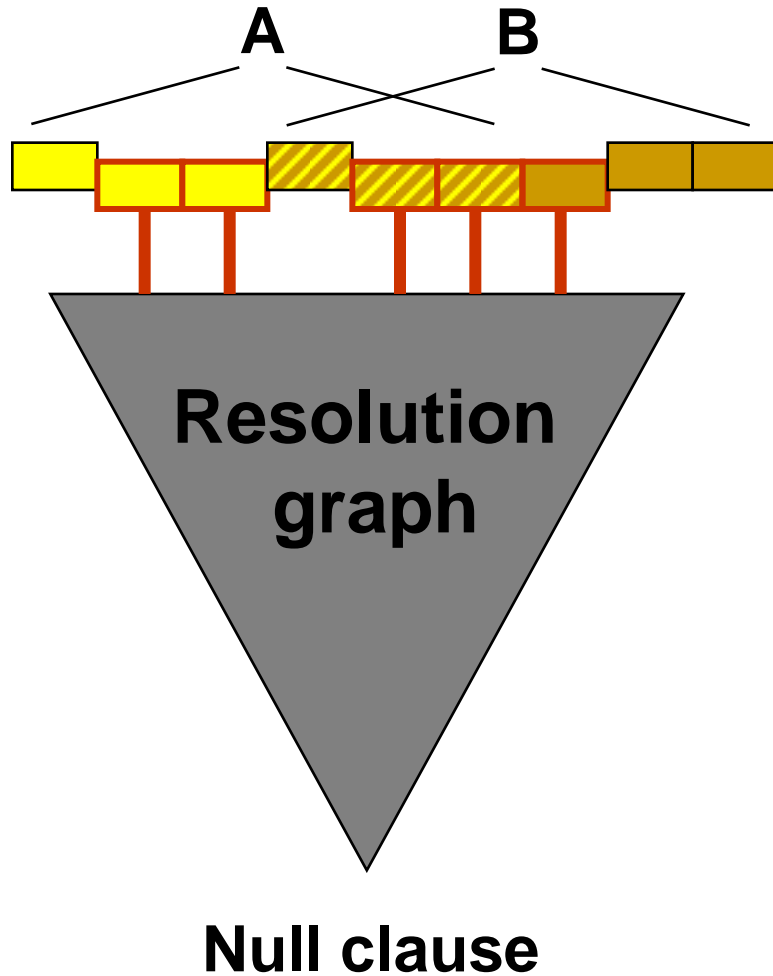
ITP from refutation proof



CNF clauses

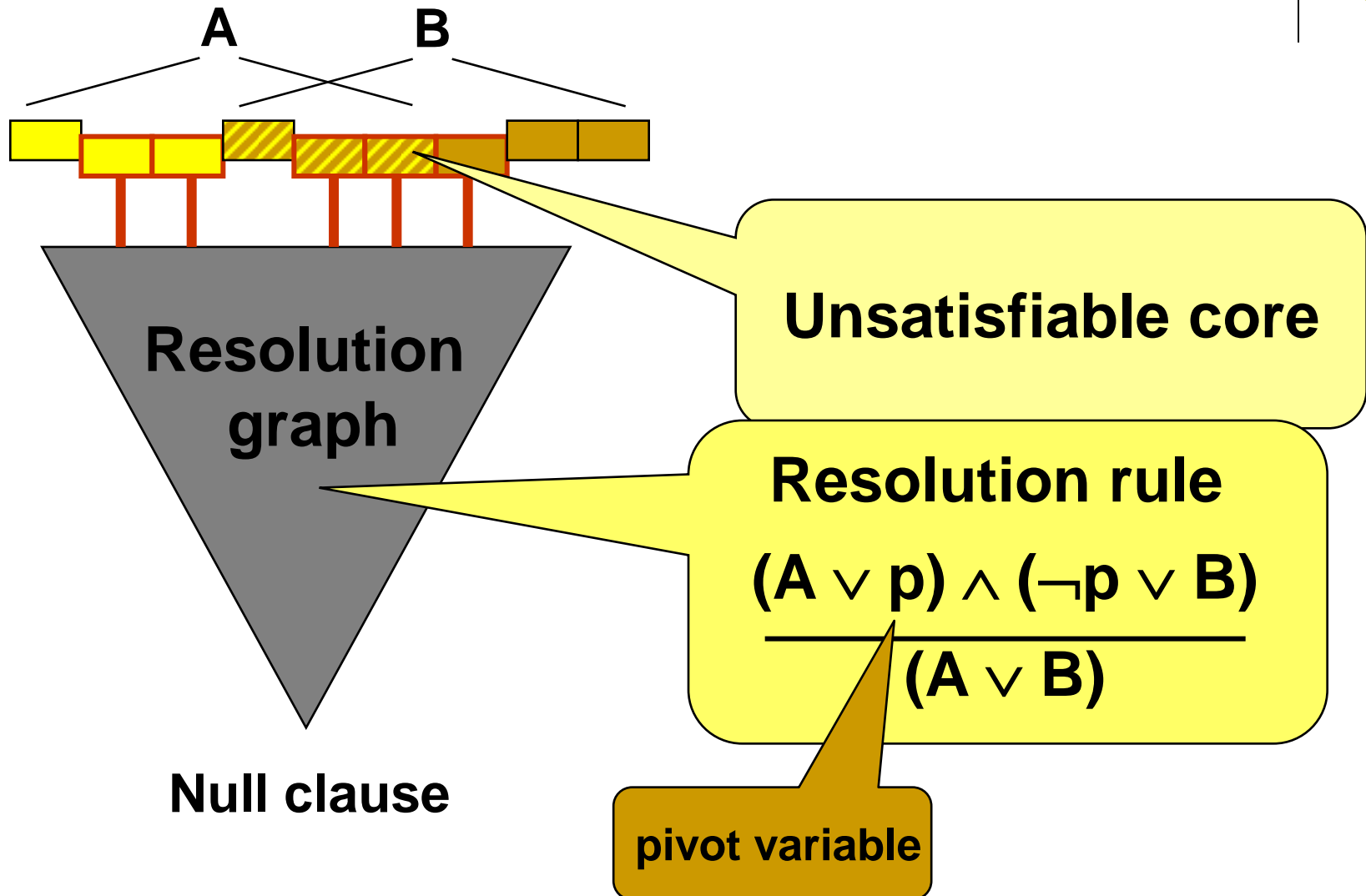
**UNSAT
problem
($A \wedge B = 0$)**

ITP from refutation proof



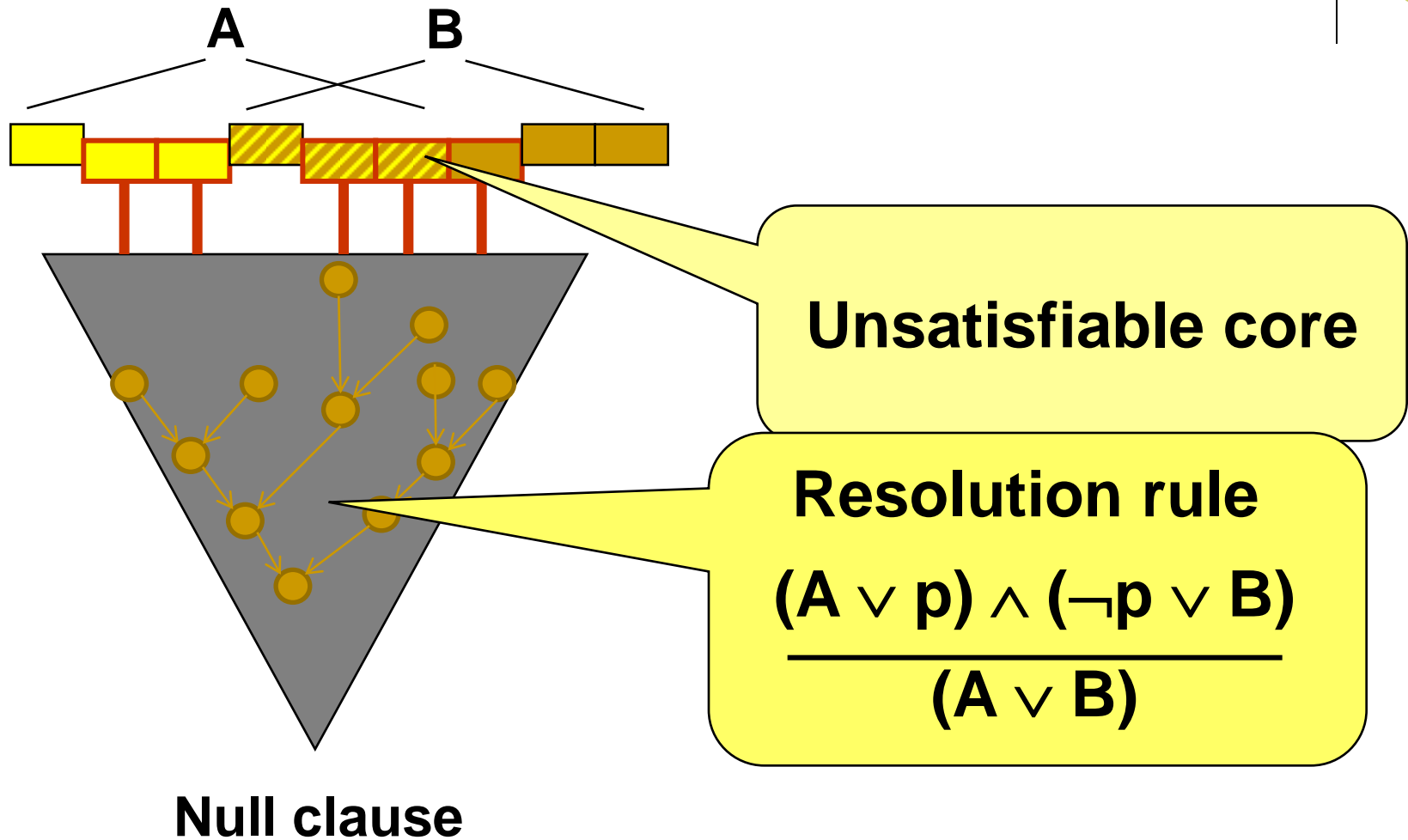


ITP from refutation proof

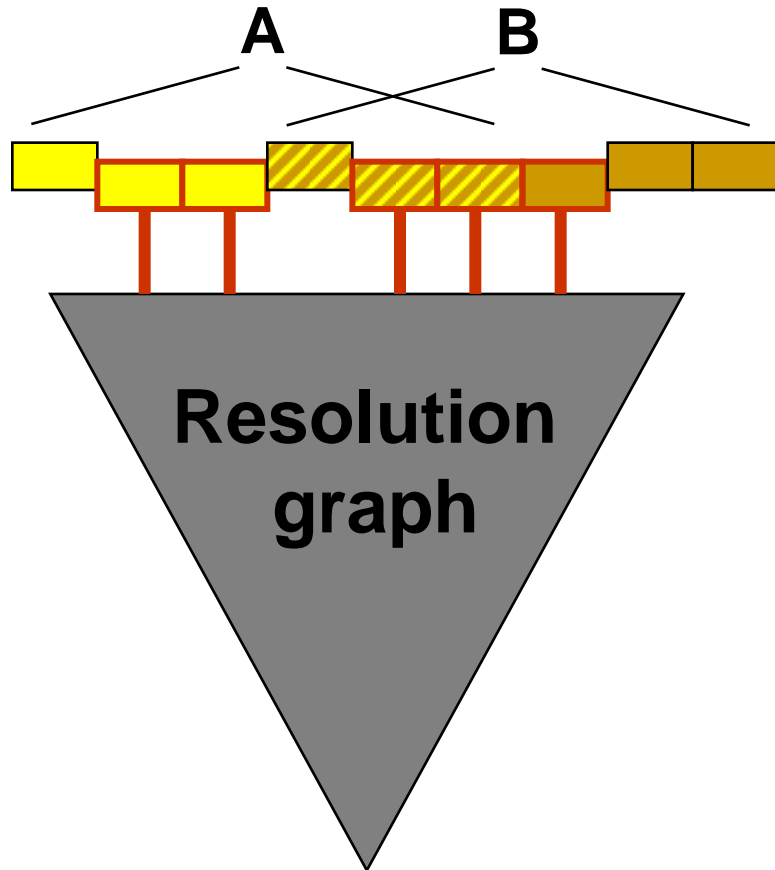




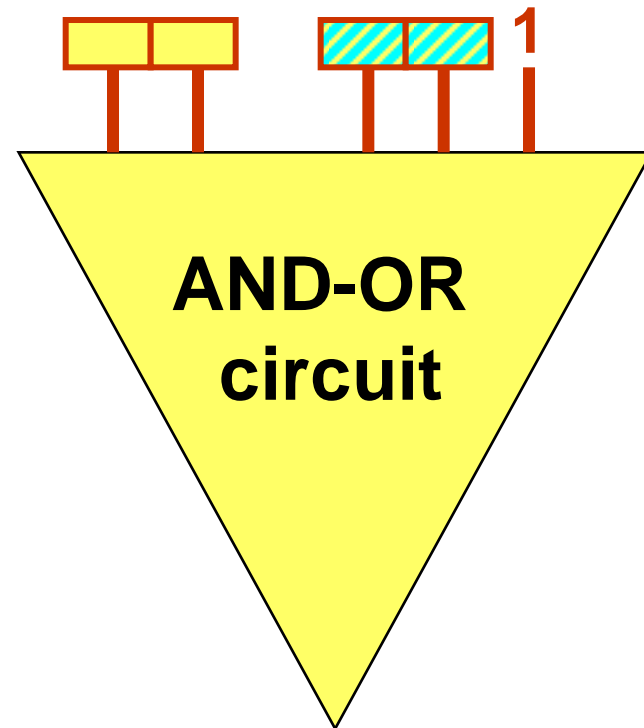
ITP from refutation proof



Interpolant from refutation proof

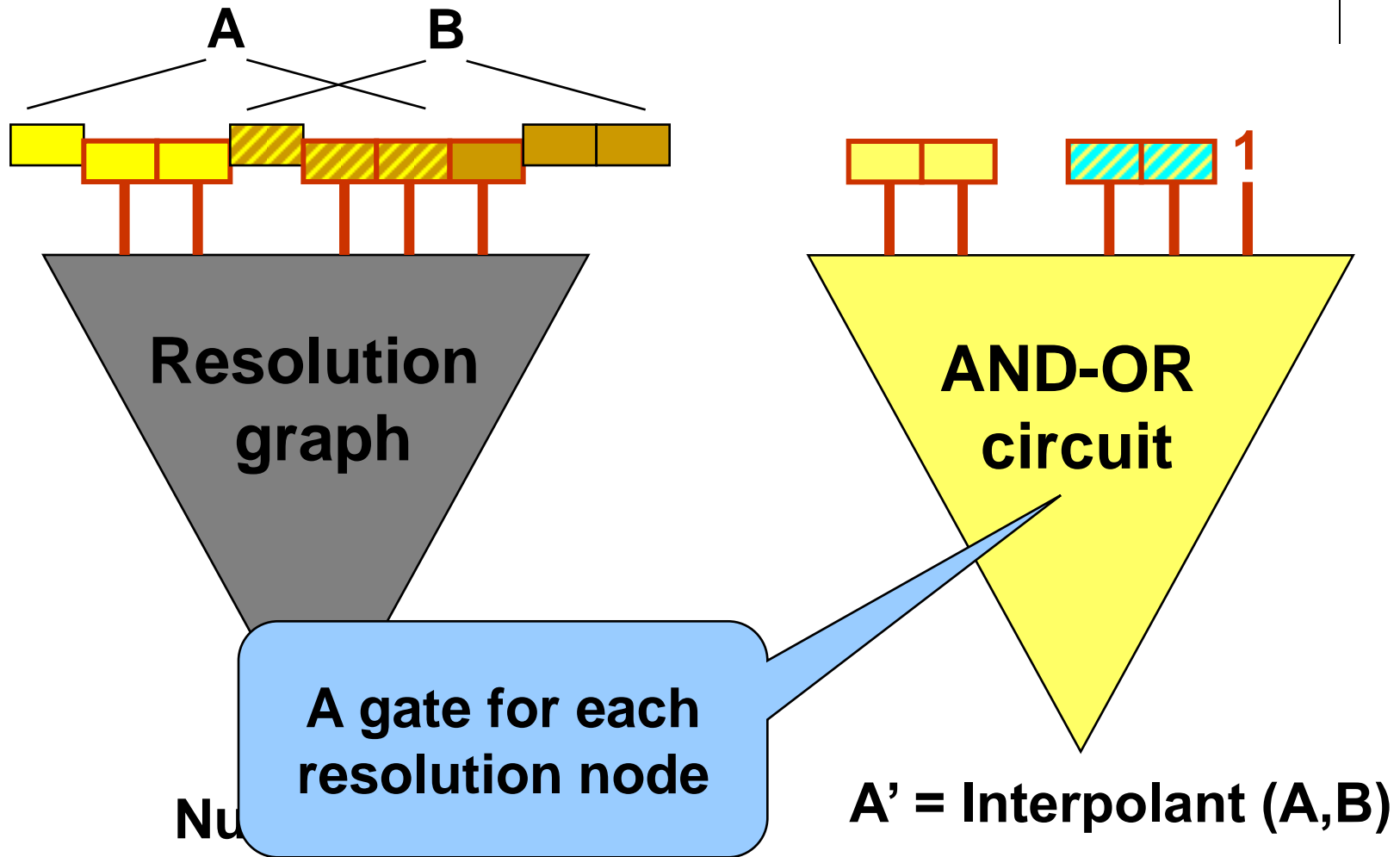


Null clause



$A' = \text{Interpolant}(A, B)$

Interpolant from refutation proof

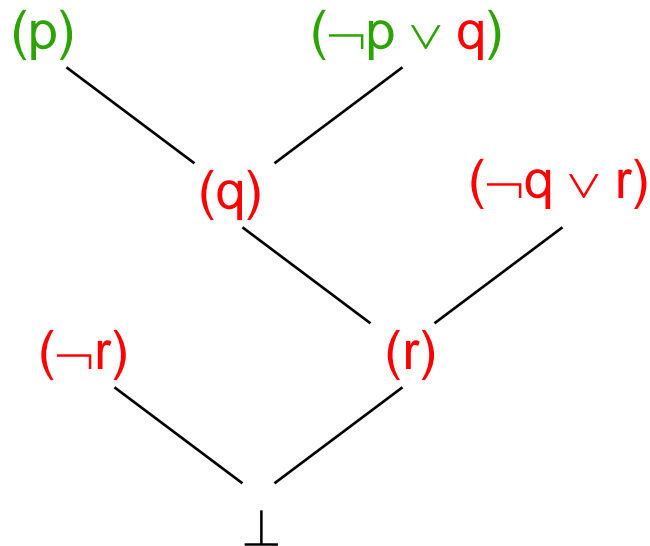




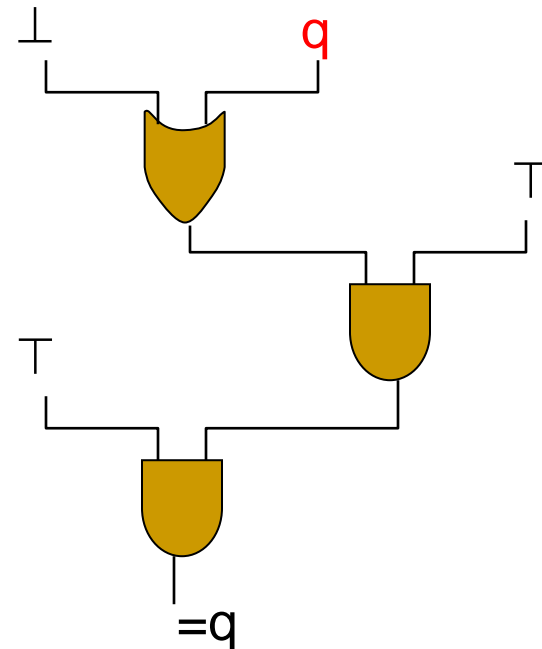
Interpolant rules

- Interpolation is a circuit that follows the *structure* of the proof

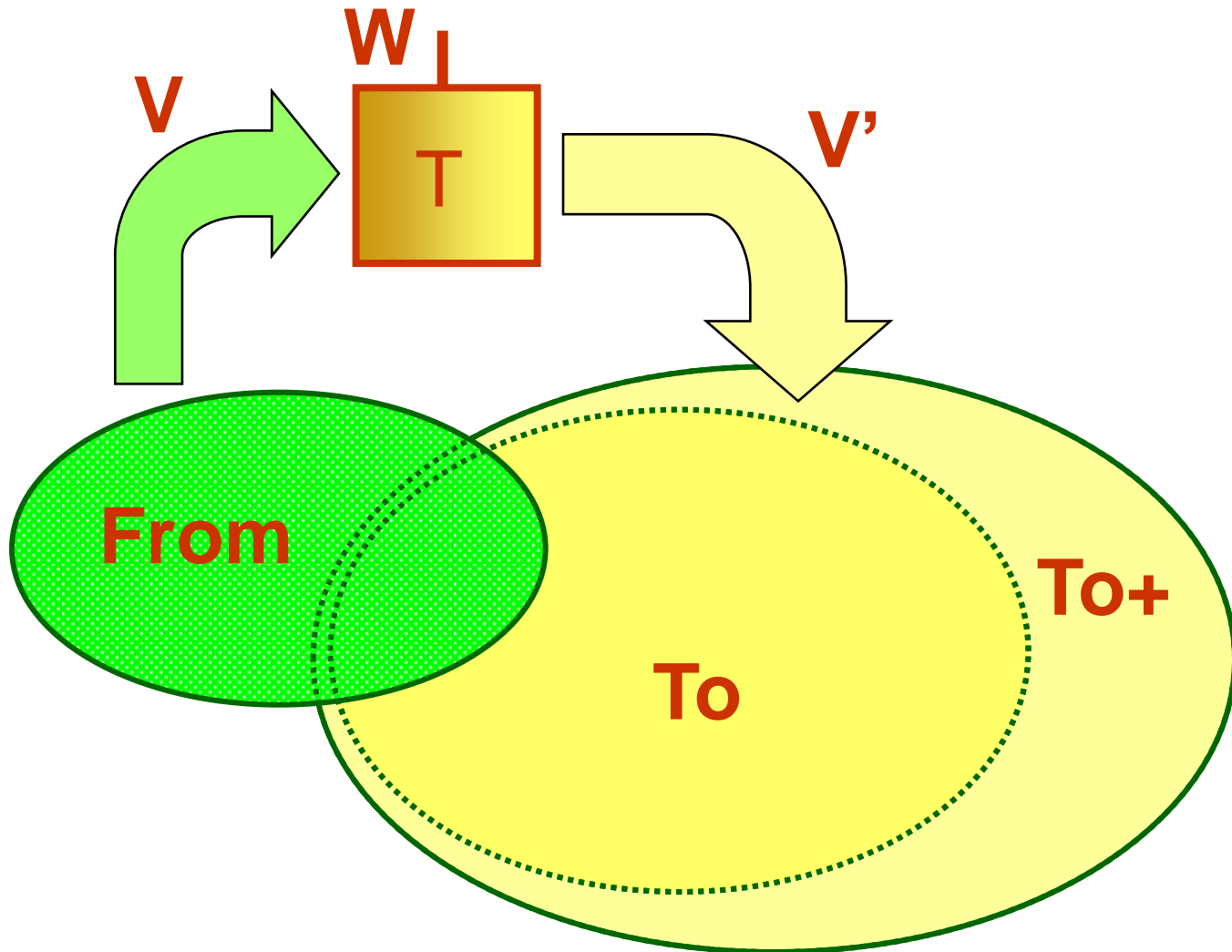
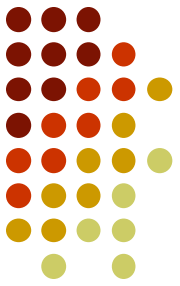
$$A = (p)(\neg p \vee q)$$



$$B = (\neg q \vee r)(\neg r)$$



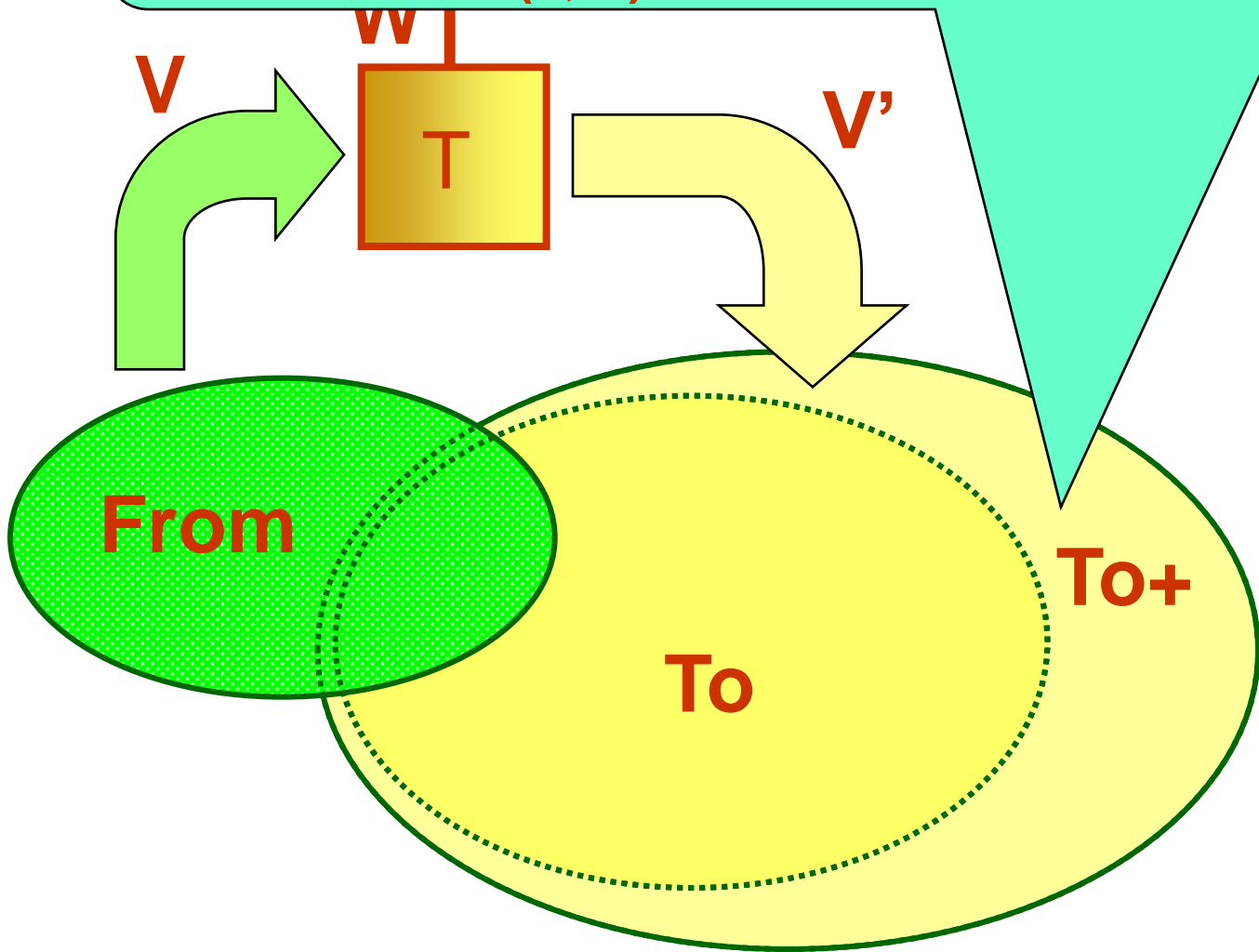
Image+



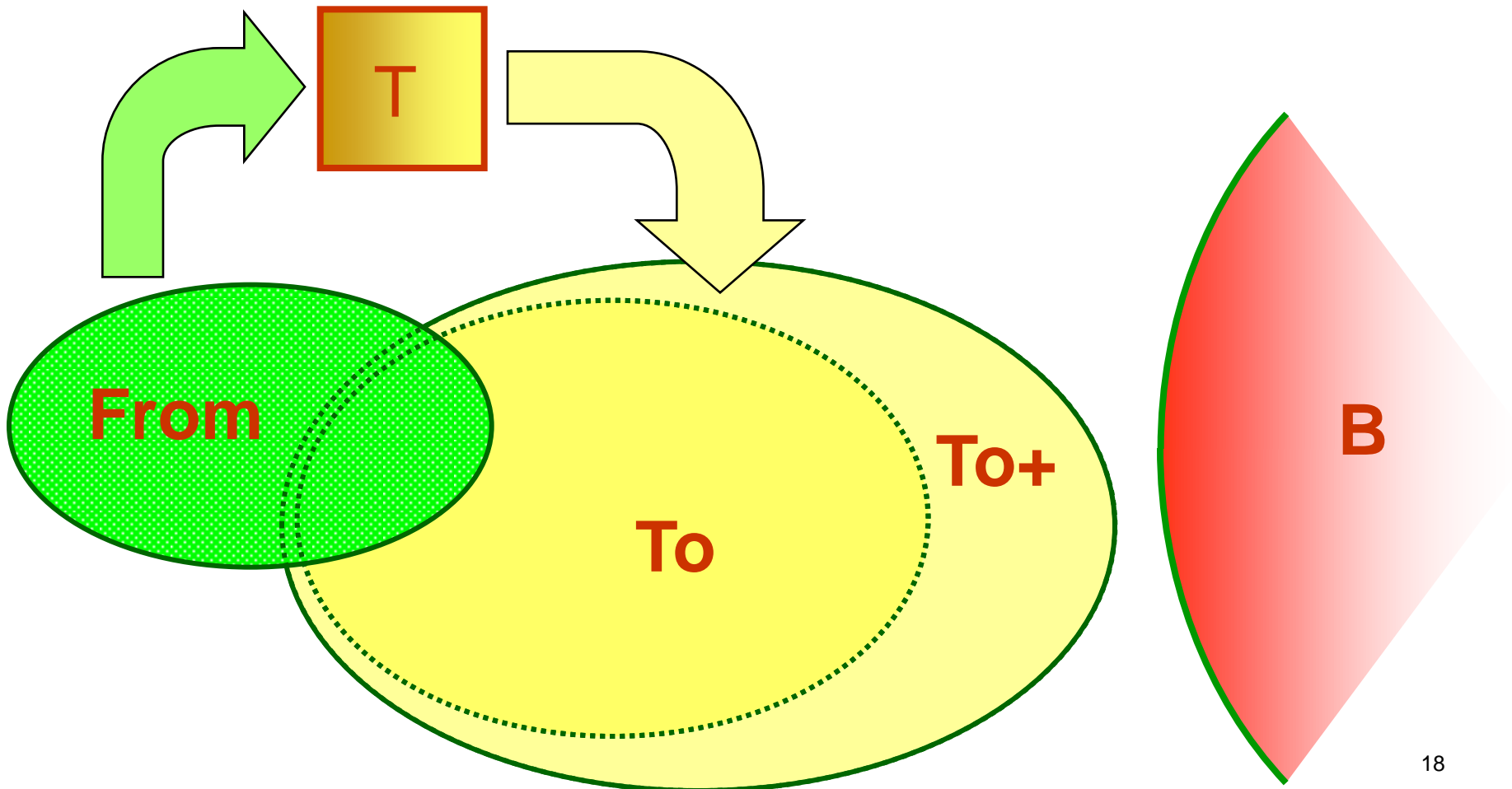


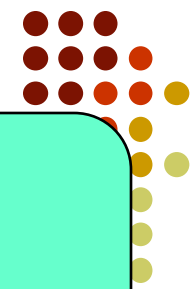
Image

$$\text{To+}(V') = \text{IMG+}(\text{From}, T) = \text{Approx}(\exists_{(v,w)} \text{From}(v) \wedge T(v, w, V'))$$



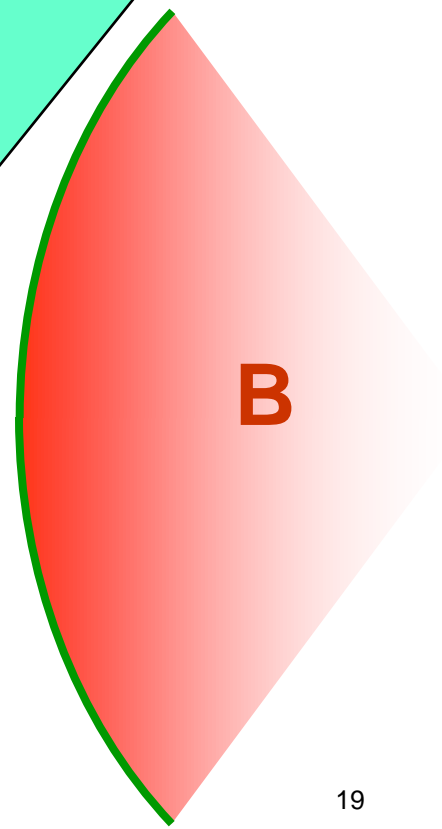
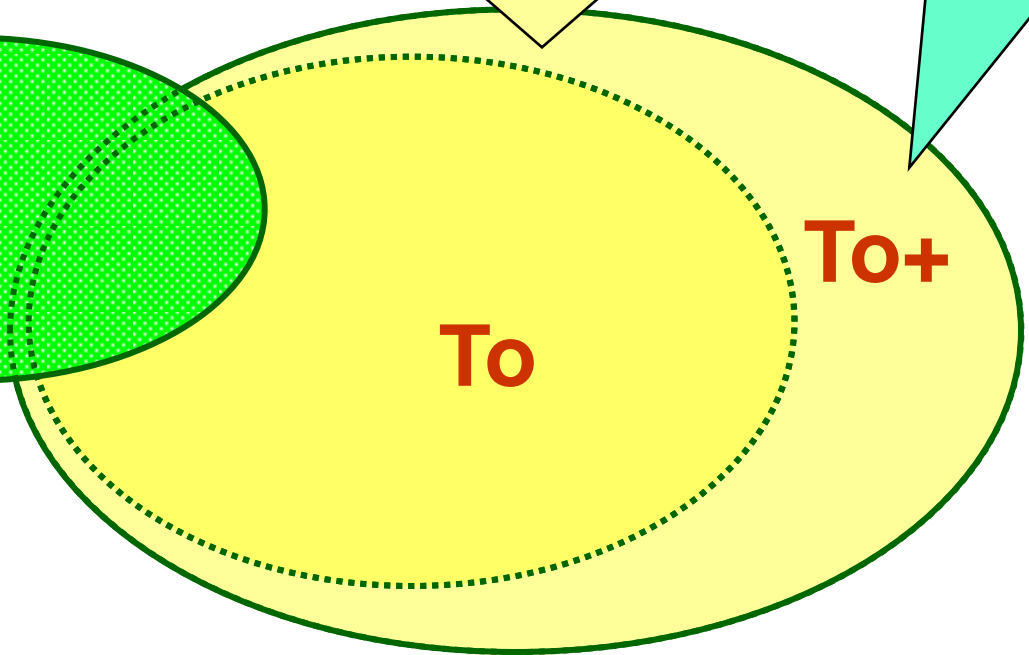
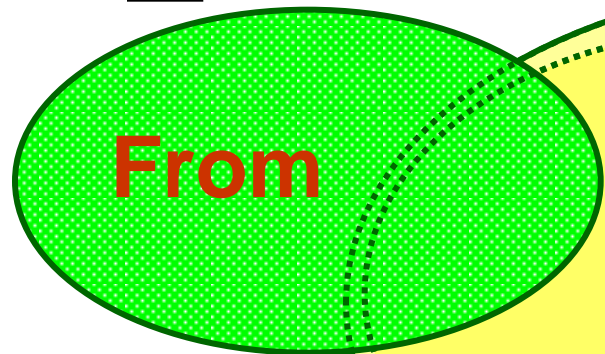
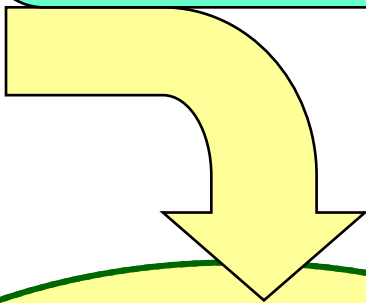
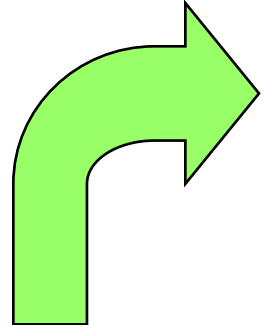
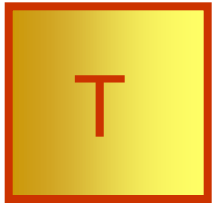
Adequate Image+





Adequate I

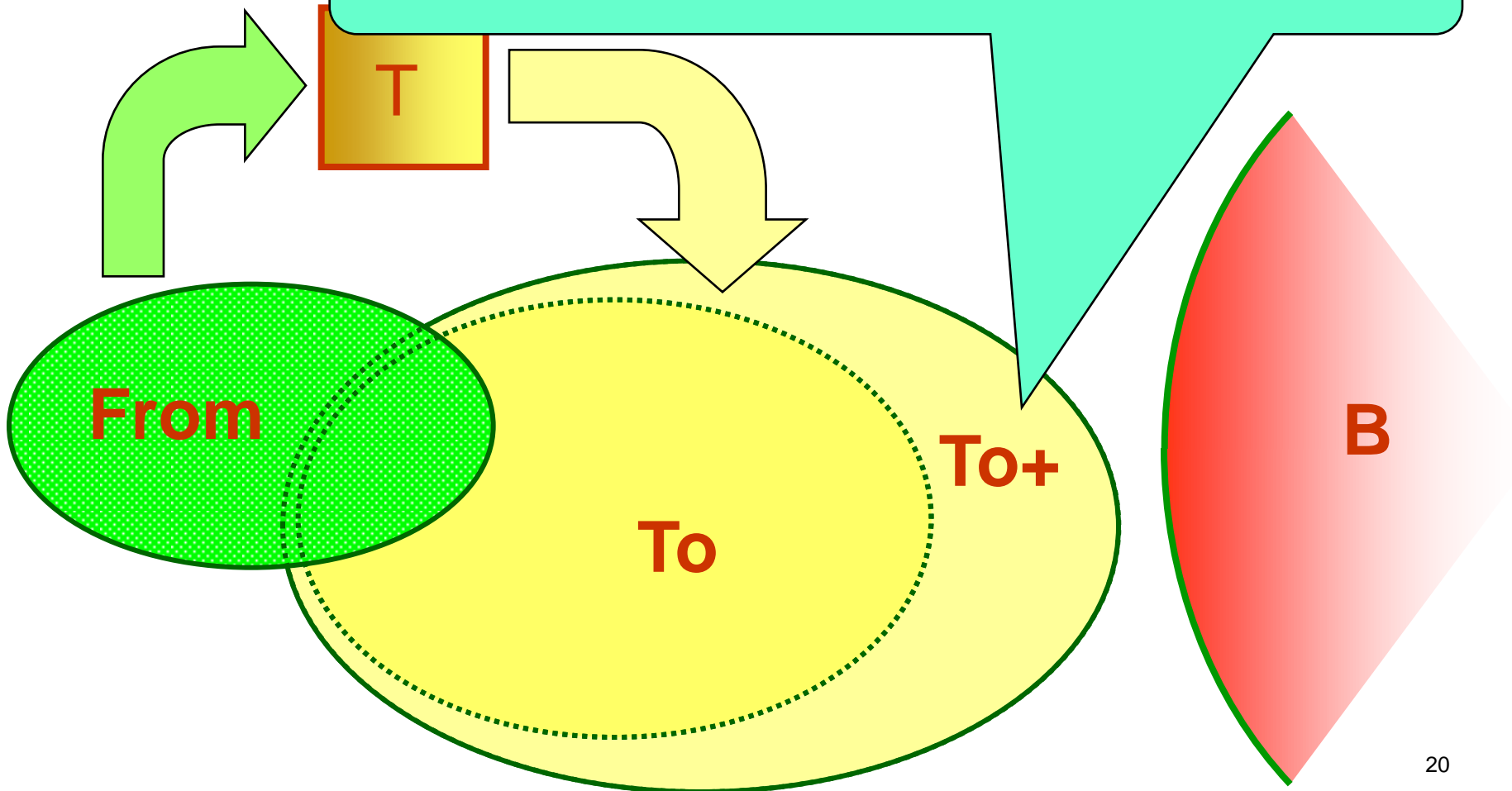
To+ adequate w.r.t. B
➤ if To outside B
➤ then To+ outside B



Adequate Image+ by Interpolant



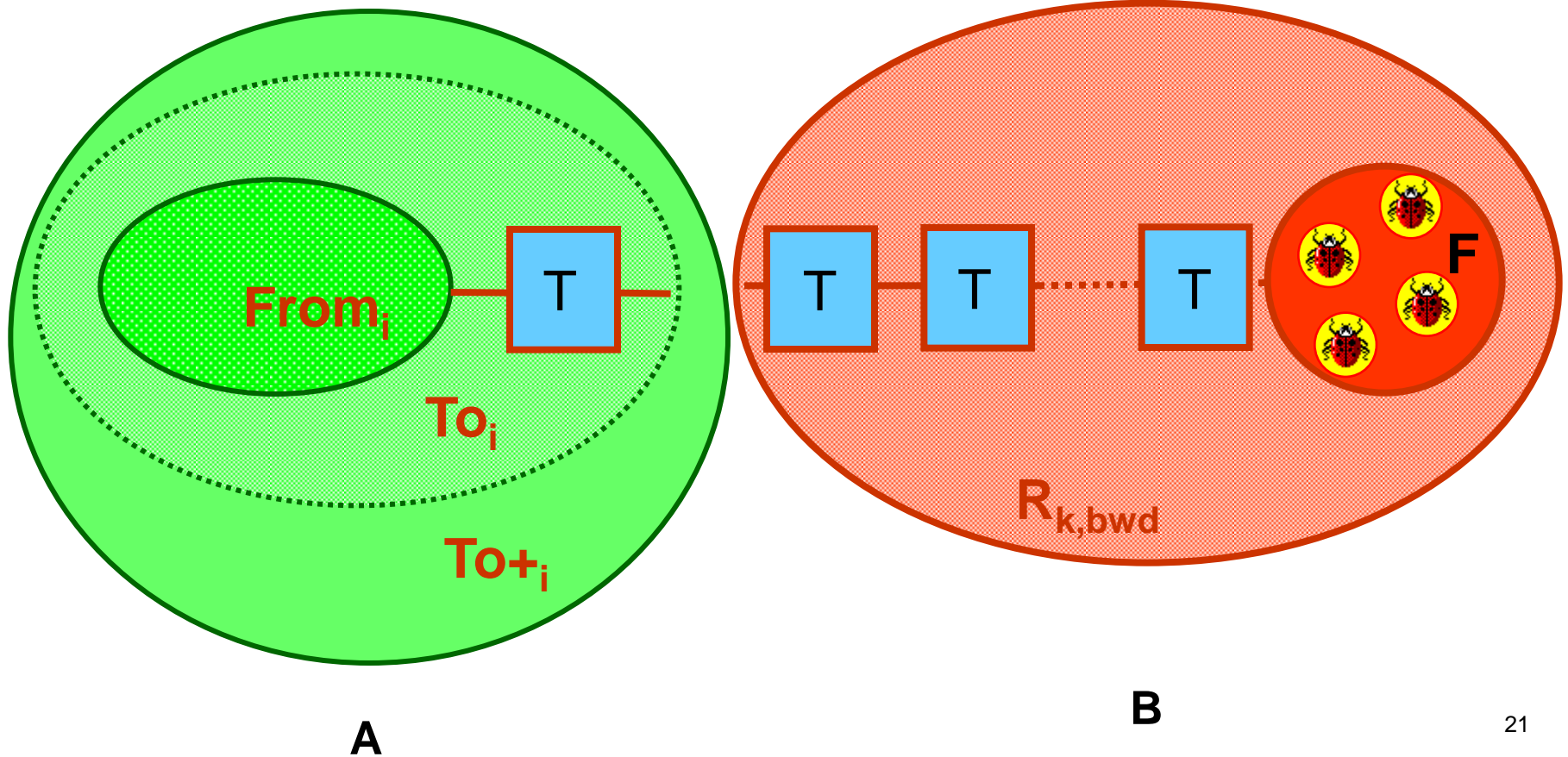
$To+ = \text{interpolant}(\text{From} \wedge T, B)$



ITP



Standard ITP: to_{+i} computed from appr. $From_i$

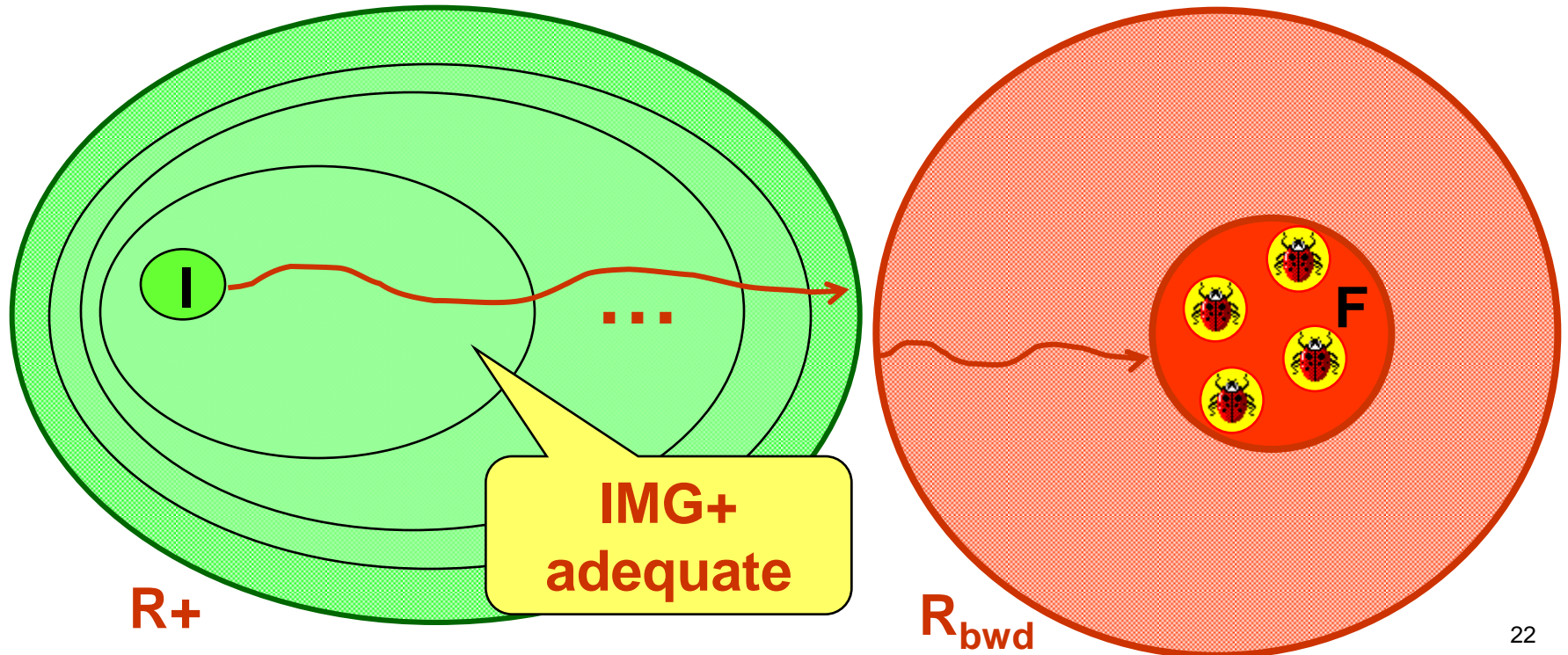




Why use adequate IMG+ ?

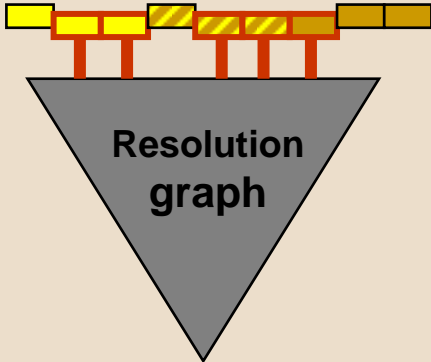
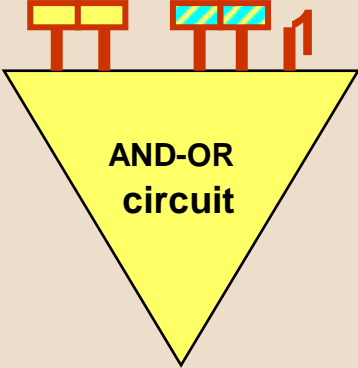
⇒ FWD *approximate* reachable states

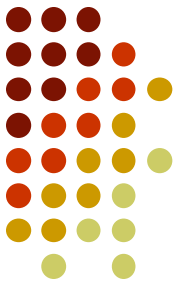
- computed by adequate IMG+
- do not intersect BWD reachable states



ITP compaction



Proof reduction	ITP circuit compaction	
 <p>Resolution graph</p>	 <p>AND-OR circuit</p>	
Alternative proofs <ul style="list-style-type: none">• different resolution schemes	BDD/SAT sweeping	Const propagation
Equivalent proofs <ul style="list-style-type: none">• redundancy removal	ODC	Refactor rewrite



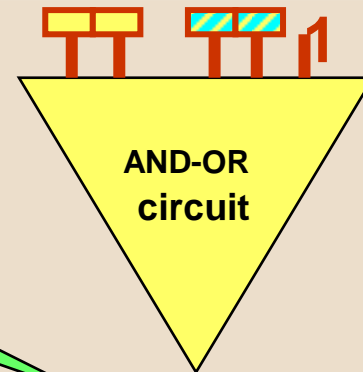
ITP compaction

Problem #1:

- SCALABILITY**

graph

ITP circuit compaction



Alternative proofs

- different resolution schemes

BDD/SAT
sweeping

Const
propagation

Equivalent proofs

- redundancy removal

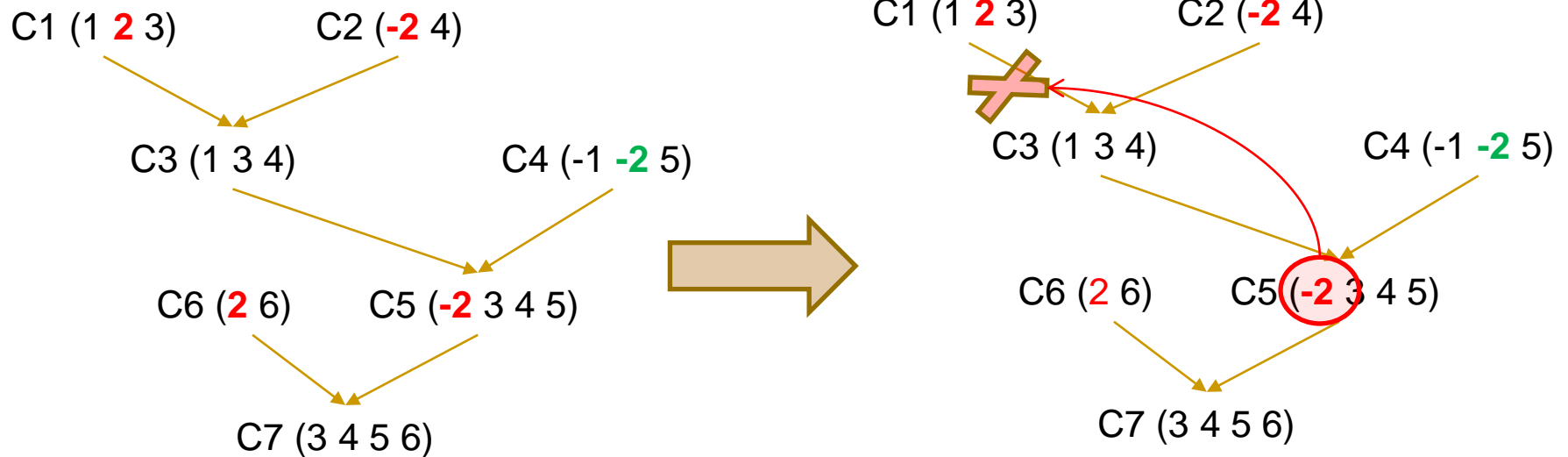
ODC

Refactor
rewrite



Proof reduction

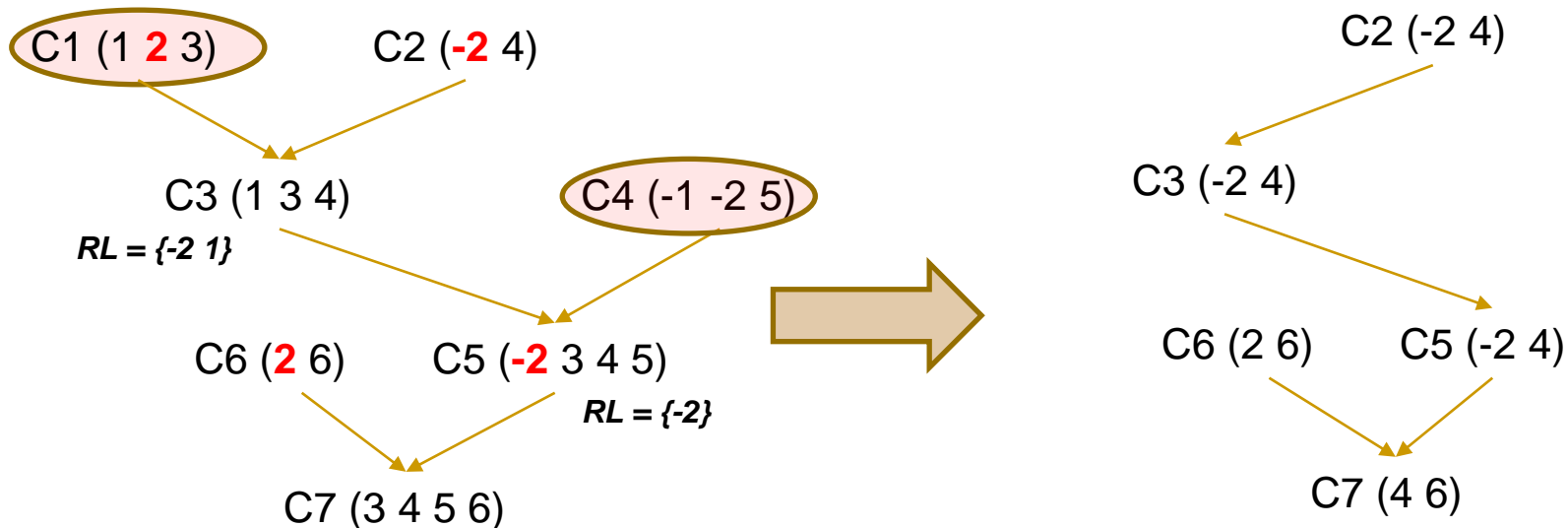
- Recycle-pivots [Bar-Inal & al. HVC08]





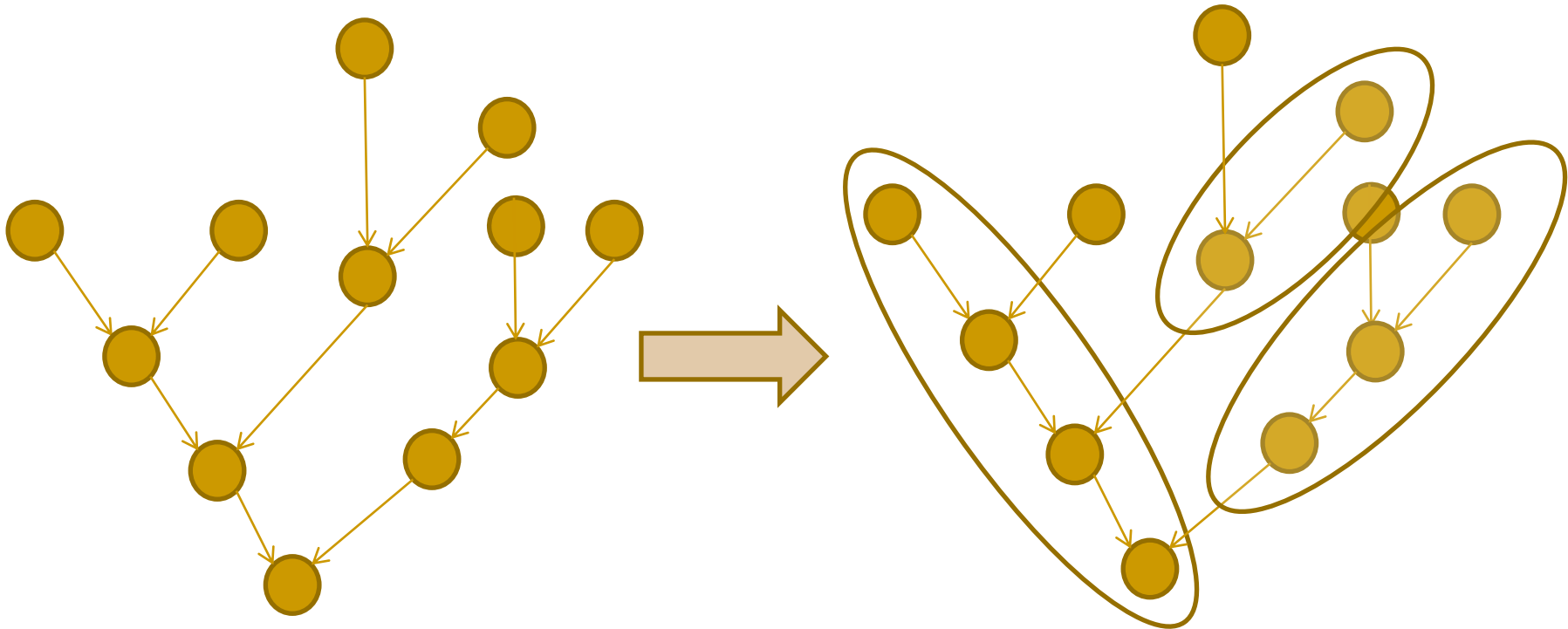
Proof reduction

- Recycle-pivots + restruct proof [Bar-Inal & al. HVC08]

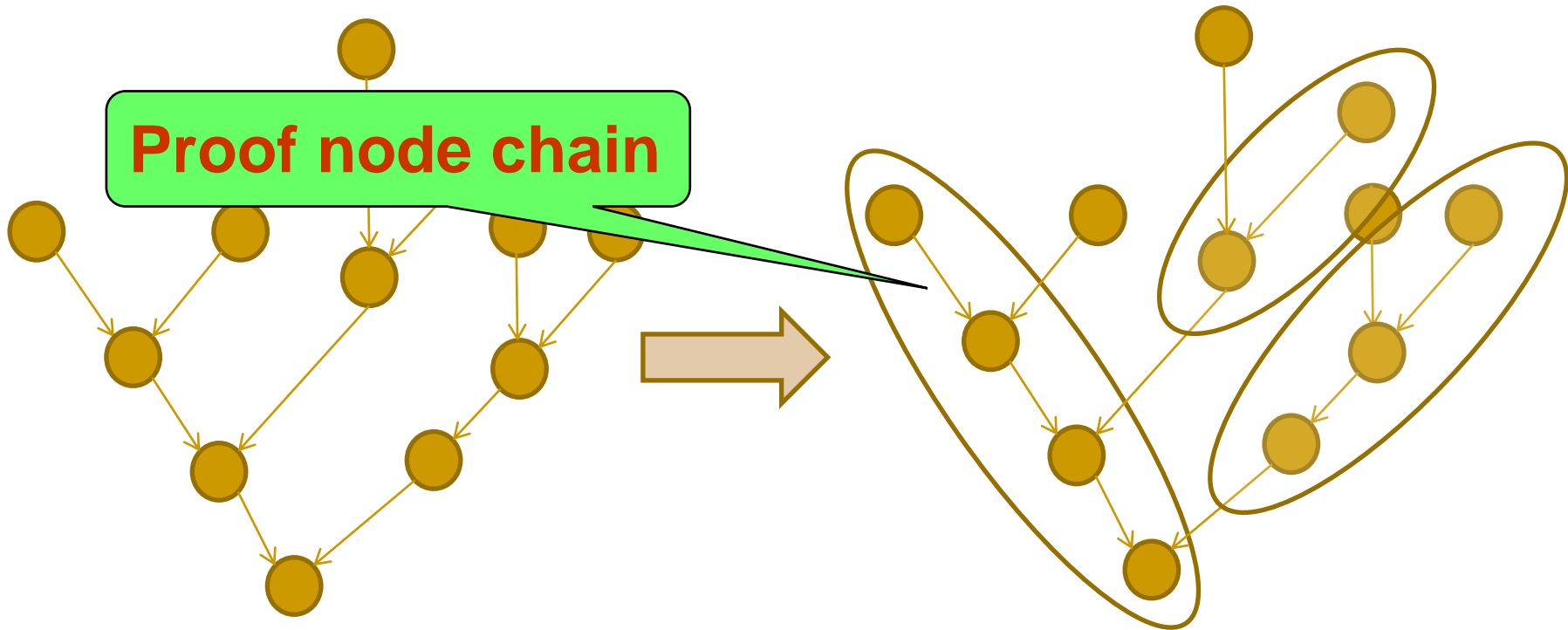


RL denotes the Removable-Literals

Our Contribution: exploit proof *topology*



Our Contribution: exploit proof topology

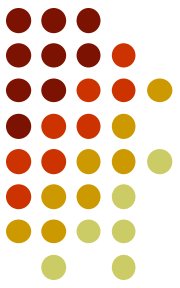


- Simpler data structure for proof reduction algorithms and further techniques



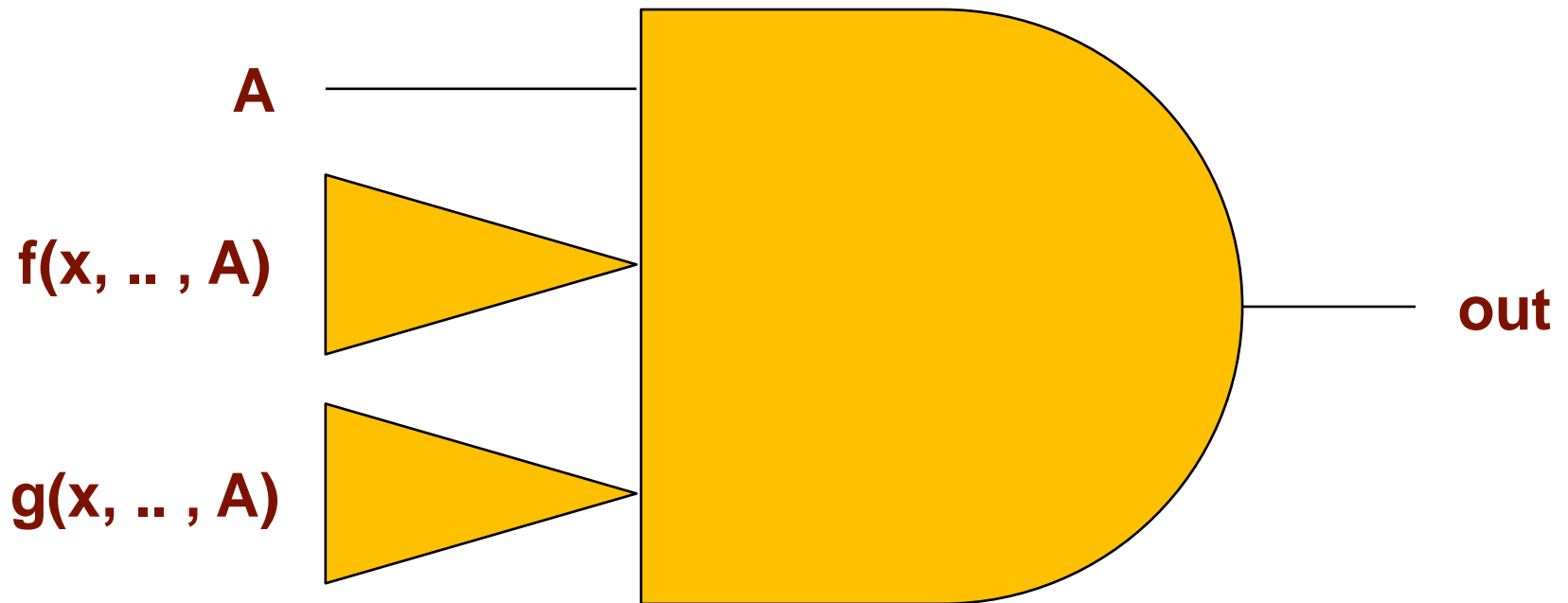
ITP Circuit Compaction

- Logic synthesis manipulations on the proof
 - Constant propagation
 - BDD-based sweeping (for equivalences)
 - Observability Don't Care (lightweight)
- Proof into AIG
 - ODC (lightweight)
 - Logic synthesis
 - rewrite / refactor, using *ABC* tool
 - AIG balance
 - ITE-based decomposition (iff necessary)



Observability don't care

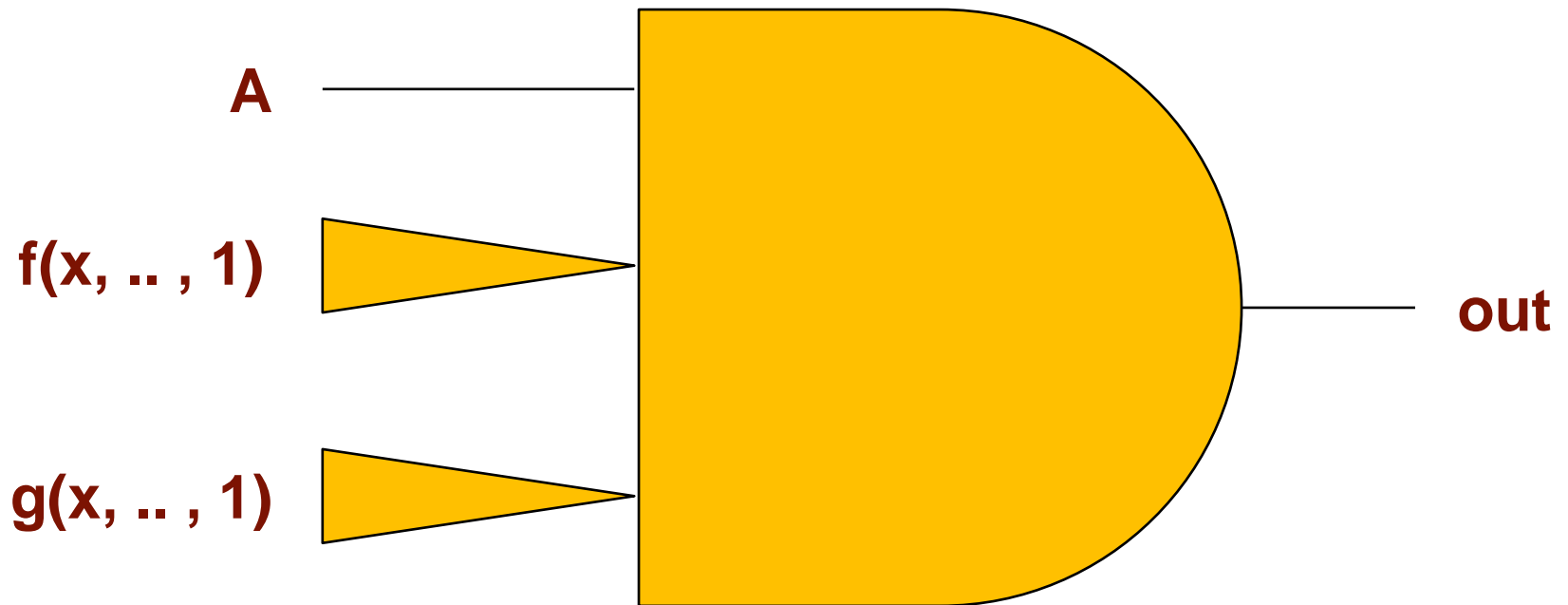
- If $A == 0 \rightarrow \text{out} = 0$; no matters $f(\cdot)$ or $g(\cdot)$
 - don't-care set



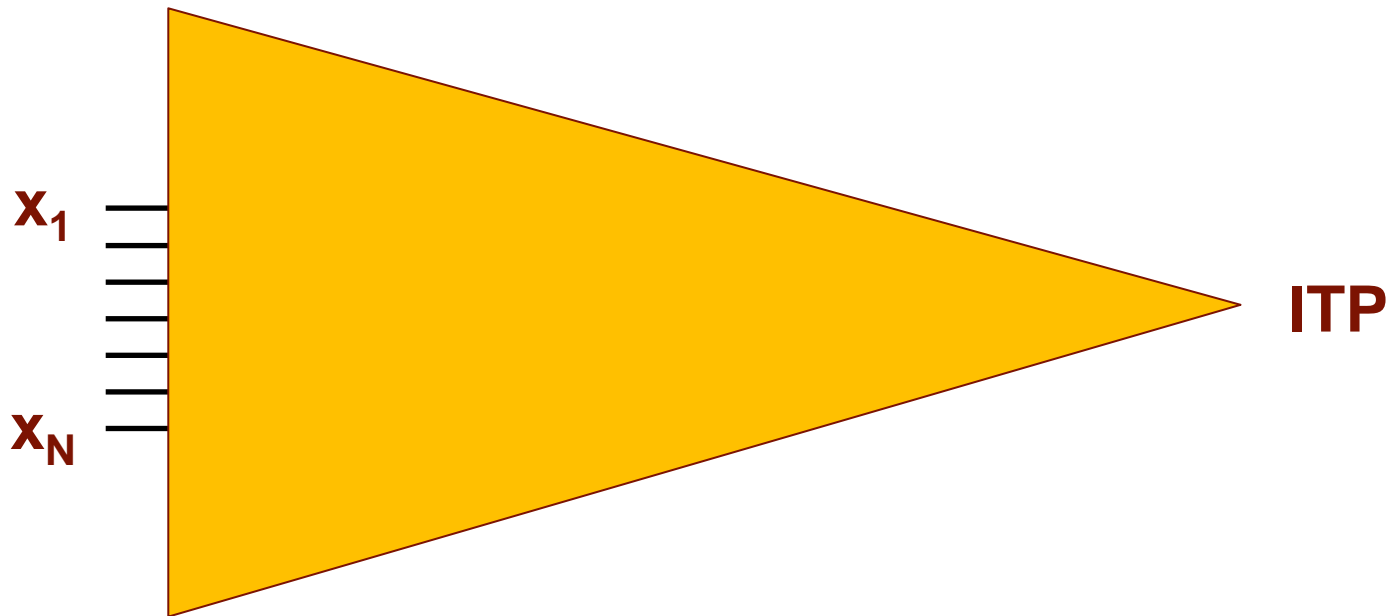


Observability don't care

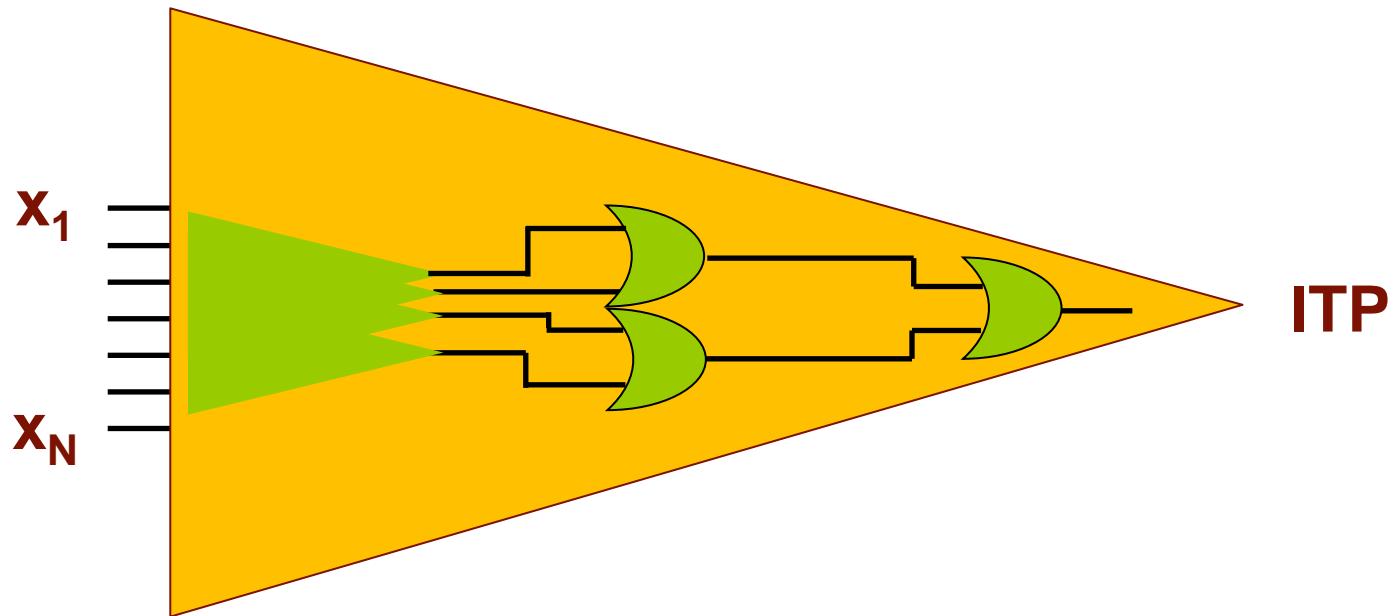
- If $A == 1 \rightarrow f(.)$ and $g(.)$ can be simplified
 - care set



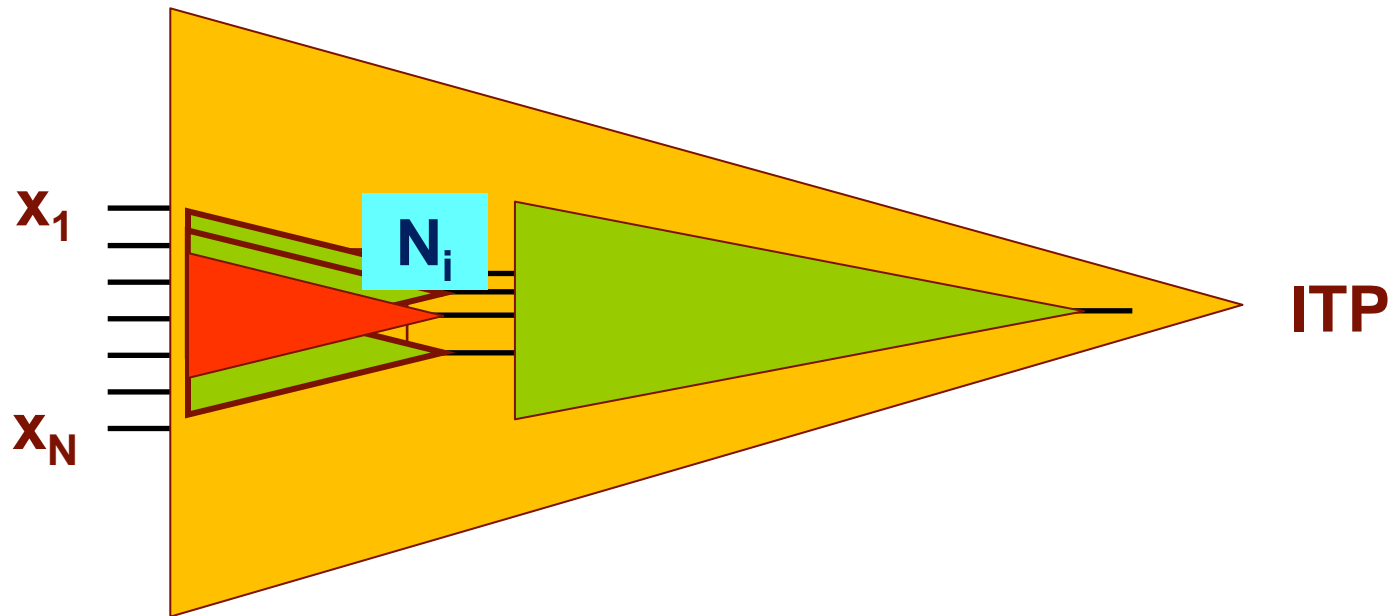
ITP ITE decomposition



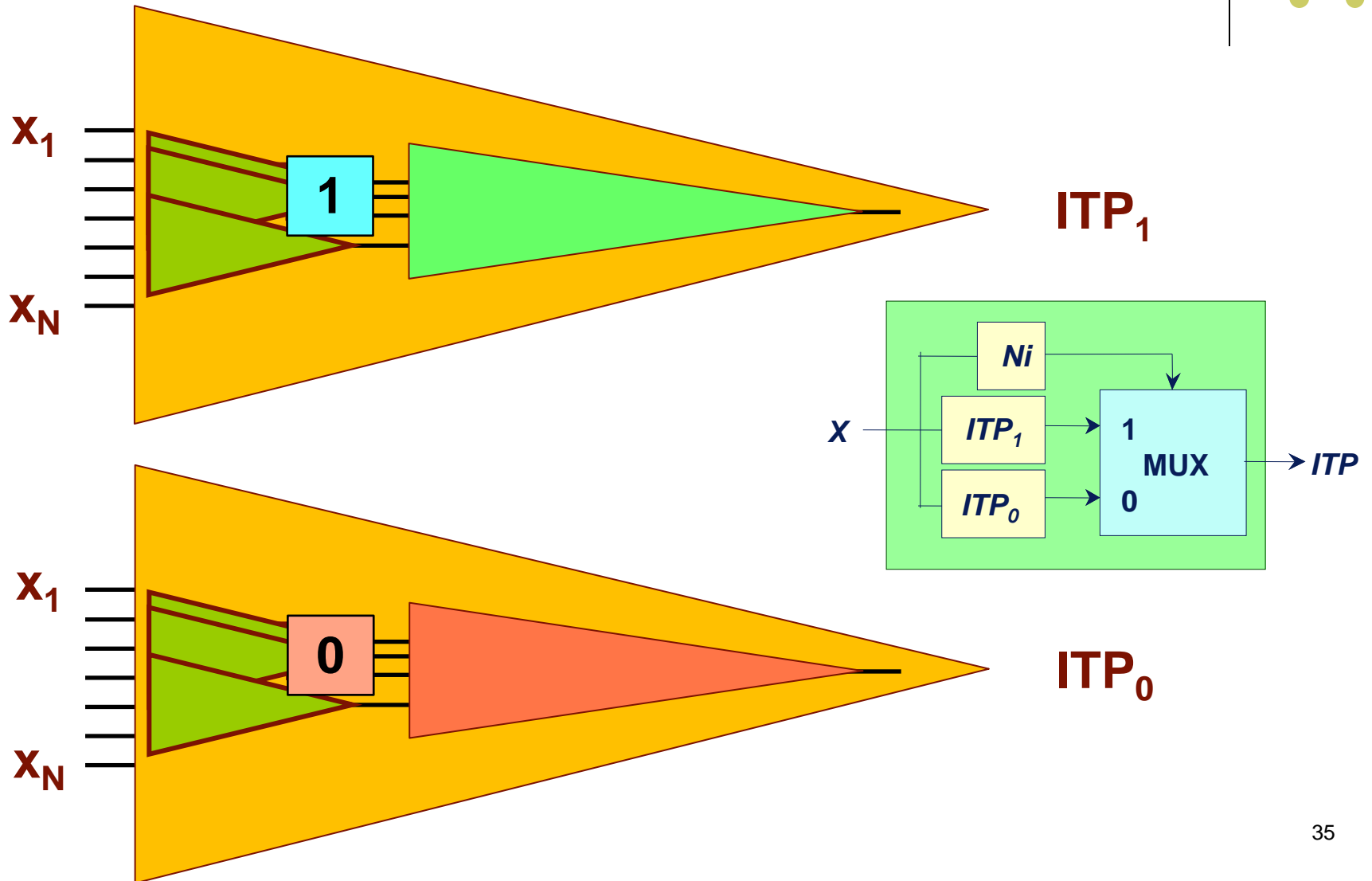
ITP ITE decomposition



ITP ITE decomposition



ITP ITE decomposition





Ad-Hoc ITP compaction

AigteDecomp (ITP)

if (max recursions || |ITP| < th)

standardLogicSynth (ITP)

do

search node N_i with highest FO

ITE(N_i, ITP_1, ITP_0) //compute cofactors; equals to ITP

if (*accept* (**ITE** decomp)) //size-based heuristic

AigteDecomp (N_i)

AigteDecomp (ITP_1)

AigteDecomp (ITP_0)

ITP = ITE(N_i, ITP_1, ITP_0)

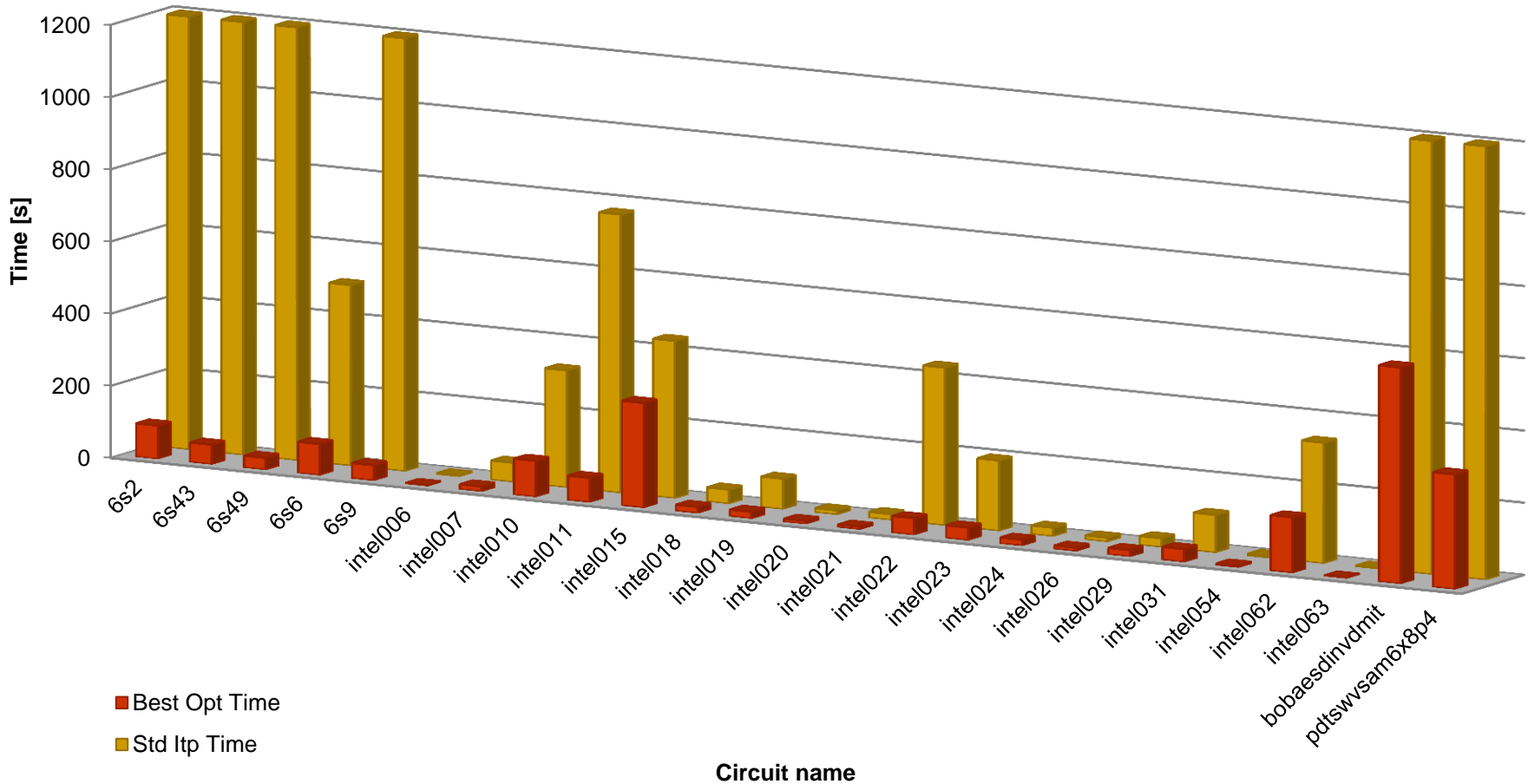
while max try reached



Experimental results

- Framework: *PdTrav*
 - State-of-the-art academic Model Checker
 - HWMCC '07 to '12
 - Ranked 1st at 2010 Model Checking Competition – UNSAT category
- ITP compaction => better MC runs
- Experience on IBM & Intel benchmarks

Experimental results





Conclusions

- ITP-based MC heavily relies on scalability, i.e. ability to compact ITPs
- We developed effective techniques to compact ITPs.
 - *Scalable techniques, applied incrementally*
- Best suited as a second engine
 - Hard-to-prove properties (hard for IC3)
 - Explosion of standard interpolation
 - Can afford extra time (for memory)



Thank you!