

# Assumption-Based Runtime Verification with Partial Observability and Resets

(RV 2019 regular paper)

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# Outline

1 The idea

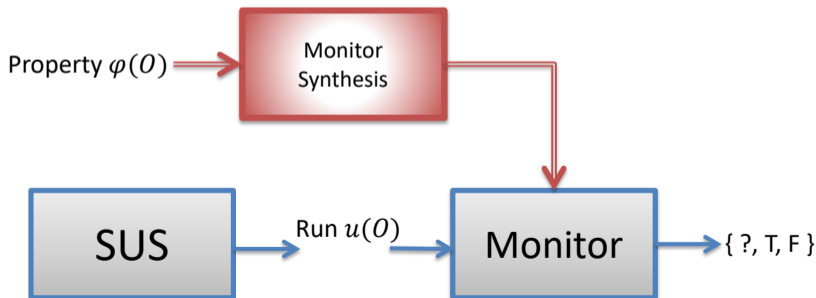
2 The definition

3 The algorithm

4 The tests

## Runtime Verification (RV)

- A lightweight verification technique providing checking if a system under scrutiny (SUS) satisfies/violates a monitoring property.
- Focus on a single *finite* trace instead of all traces from SUS.
- Has an *incremental* fashion, outputting verdicts on each input state.
- Applicable to *black box* systems where a model is not available.
- Assumes full observability usually.

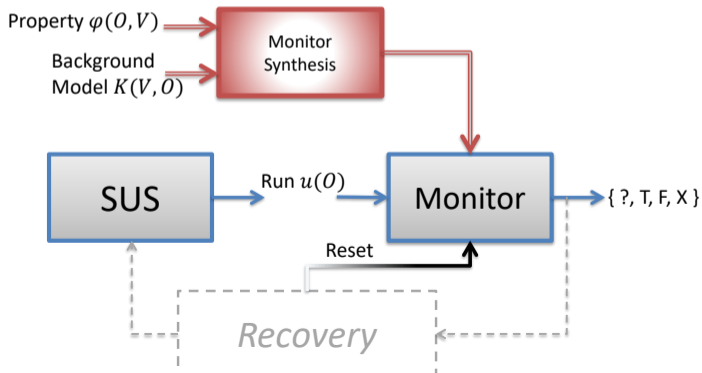


# Assumption-Based Runtime Verification

However, one *almost always* knows something about the SUS, e.g.

- Models produced during the system design;
- Interaction with system operators (i.e. people) — domain knowledge.
- Mathematics/physical principles, e.g.  $\varphi = (i \leq 5) \cup (i > 10)$ .

These knowledge may be leveraged to get *better* monitors.



## Resettable Monitors

- Traditionally the monitor only evaluates  $\llbracket u \models \varphi \rrbracket (= \llbracket u, 0 \models \varphi \rrbracket)$ ;
- The resettable monitor takes as input some *reset* signals that change the reference time of evaluating monitor properties, e.g. from  $\llbracket u, i \models \varphi \rrbracket$  to  $\llbracket u, j \models \varphi \rrbracket$  ( $j > i$ );
- The execution history of SUS is preserved during resets, possible impacts to monitoring outputs:
  - ① Under assumptions, the belief states after resets may be different with initial belief states;
  - ② With past operators, historical inputs may change the initial evaluation of a monitoring property.

### The Motivation

- ① Monotonic monitors: still meaningful after reaching conclusive verdicts (then being reset).
- ② Monitoring Past-Time LTL (to be explained).

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# (Propositional) Linear Temporal Logic

## Syntax ( $p \in AP$ )

$$\varphi ::= \text{true} \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid X\varphi \mid \varphi U \varphi \mid Y\varphi \mid \varphi S \varphi$$

- X stands for *next*, U for *until*, Y for *previous*, S for *since*.
- logical constants and operators like false,  $\wedge$ ,  $\rightarrow$  and  $\leftrightarrow$  are used as syntactic sugars with the standard meaning.
- Abbreviations:

$F\varphi \doteq \text{true} U \varphi$ (eventually),

$G\varphi \doteq \neg F \neg\varphi$ (globally),

$O\varphi \doteq \text{true} S \varphi$ (once),

$H\varphi \doteq \neg O \neg\varphi$ (historically).

## Recall: $LTL_3$ semantics

- Three-valued semantics of LTL formula  $\varphi$  over a finite word  $u \in \Sigma^*$ :

$$\llbracket u, i \models \varphi \rrbracket_3 = \begin{cases} \top, & \text{if } \forall w \in \Sigma^\omega. u \cdot w, i \models \varphi, \\ \perp, & \text{if } \forall w \in \Sigma^\omega. u \cdot w, i \not\models \varphi, \\ ?, & \text{otherwise .} \end{cases}$$

with  $\llbracket u \models \varphi \rrbracket_3$  denoting  $\llbracket u, 0 \models \varphi \rrbracket_3$ .

- $\llbracket u \models \varphi \rrbracket_3 = \top/\perp$  if all extensions of  $u$  satisfy/violate  $\varphi$ ;
- Monitor construction:<sup>1</sup>

$$\mathcal{M}_\varphi(u) = \llbracket u \models \varphi \rrbracket_3 .$$

<sup>1</sup>A. Bauer, M. Leucker, and C. Schallhart. *Runtime Verification for LTL and TLTL*. (2011)



## ABRV-LTL semantics

Let  $K \doteq \langle V_K, \Theta_K, \rho_K, \mathcal{J}_K \rangle$  be an FKS,  $\varphi$  be an LTL formula built from  $AP$ . Let  $\psi(O) \in \Psi(O)^*$  be a finite sequence of Boolean formulae over  $O \subseteq V_K \cup AP$ . We also define

$$\mathcal{L}^K(\psi(O)) \doteq \{w \in \mathcal{L}(K) \mid \forall i. i < |\psi(O)| \Rightarrow w_i(V_K \cup AP) \models \psi_i(O)\}$$

to be the set of runs in  $K$  which are compatible with  $\psi(O)$ .

### Definition

The ABRV-LTL semantics of  $\varphi$  over  $\psi(O)$  under  $K$  is defined as

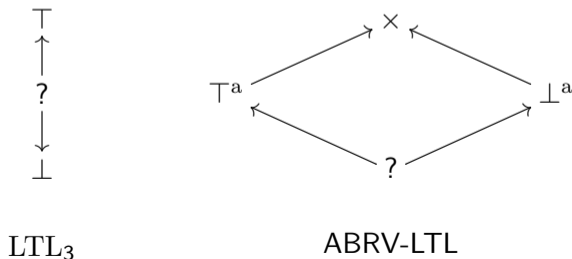
$$\llbracket \psi(O), i \models \varphi \rrbracket_4^K \doteq \begin{cases} \times, & \text{if } \mathcal{L}^K(\psi(O)) = \emptyset \\ \top^a, & \text{if } \mathcal{L}^K(\psi(O)) \neq \emptyset \wedge \forall w \in \mathcal{L}^K(\psi(O)). w, i \models \varphi \\ \perp^a, & \text{if } \mathcal{L}^K(\psi(O)) \neq \emptyset \wedge \forall w \in \mathcal{L}^K(\psi(O)). w, i \models \neg\varphi \\ ?, & \text{otherwise .} \end{cases}$$

## ABRV-LTL verdicts (and the lattice)

$\mathbb{B}_4 \doteq \{\top^a, \perp^a, ?, \times\}$ :

- *conclusive true* ( $\top^a$ ) (or *true under assumption*)
- *conclusive false* ( $\perp^a$ ) (or *false under assumption*)
- *inconclusive* (?)
- *out-of-model* ( $\times$ )

The lattice



# The ABRV Framework

- The input is enriched with resets:  $u \in (\Psi(O) \times \mathbb{B})^*$ .
- Each input state is a pair of an observation and a Boolean representing the reset;
- Given a property  $\phi$  and an assumption  $K$ , the problem of *Assumption-based Runtime Verification (ABRV)* is to construct a function  $\mathcal{M}_\phi^K : (\Psi(O) \times \mathbb{B})^* \rightarrow \mathbb{B}_4$  such that

$$\mathcal{M}_\phi^K(u) = \llbracket \text{OBS}(u), \text{MRR}(u) \models \phi \rrbracket_4^K$$

where

- $\text{OBS}(\cdot)$  (observations) is the projection of  $u$  from  $\Psi(O) \times \mathbb{B}$  to  $\Psi(O)$ ,
- $\text{RES}(\cdot)$  (resets) is the projection of  $u$  (or  $u_i$ ) from  $\Psi(O) \times \mathbb{B}$  to  $\mathbb{B}$ ,
- $\text{MRR}(u)$  (the most recent reset) is the maximal  $i$  such that  $\text{RES}(u_i) = \top$ .

## Special Case: Runtime Verification of Past-Time LTL

Past-Time LTL (PtLTL)<sup>2</sup>: LTL with only past operators (Y, S).

Let  $u = s_1 s_2 \cdot s_n$  and  $u_i = s_1 s_2 \cdot s_i$ :

$$u \models_p p \Leftrightarrow p \in s_{n-1}$$

$$u \models_p Y \varphi \Leftrightarrow u_{n-1} \models_p \varphi \text{ (if } n > 1) \text{ or } u \models_p \varphi \text{ (if } n = 1)$$

$$u \models_p \varphi S \psi \Leftrightarrow u_j \models_p \psi \text{ (} 1 \leq j \leq n) \text{ and } u_i \models_p \varphi \text{ (} j < i \leq n)$$

Convert PtLTL to ABRV-LTL ( $K$  is empty)

$$\llbracket u \models_p \varphi \rrbracket = \top \Leftrightarrow \llbracket u, |u| - 1 \models \varphi \rrbracket_4^K = \top^a,$$

$$\llbracket u \models_p \varphi \rrbracket = \perp \Leftrightarrow \llbracket u, |u| - 1 \models \varphi \rrbracket_4^K = \perp^a.$$

$\text{MRR}(u) = |u|$  or  $\forall i. \text{RES}(u_i) = \top$ .

<sup>2</sup>K. Havelund and G. Roşu. [Synthesizing Monitors for Safety Properties](#).

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# Translating LTL to $\omega$ -automata (1)

## Elementary Variables

$$\begin{aligned}
 \text{el}(\text{true}) &= \emptyset, & \text{el}(\mathbf{X}\phi) &= \{\mathbf{x}_\phi\} \cup \text{el}(\phi), \\
 \text{el}(p) &= \{p\}, & \text{el}(\phi \mathbf{U} \psi) &= \{\mathbf{x}_{\phi \mathbf{U} \psi}\} \cup \text{el}(\phi) \cup \text{el}(\psi), \\
 \text{el}(\neg\phi) &= \text{el}(\phi), & \text{el}(\mathbf{Y}\phi) &= \{\mathbf{y}_\phi\} \cup \text{el}(\phi), \\
 \text{el}(\phi \vee \psi) &= \text{el}(\phi) \cup \text{el}(\psi), & \text{el}(\phi \mathbf{S} \psi) &= \{\mathbf{y}_{\phi \mathbf{S} \psi}\} \cup \text{el}(\phi) \cup \text{el}(\psi) .
 \end{aligned}$$

## Expansion Laws (for the xNF conversion)

$$\psi \mathbf{U} \phi \Leftrightarrow \phi \vee (\psi \wedge \mathbf{X}(\psi \mathbf{U} \phi)), \quad \psi \mathbf{S} \phi \Leftrightarrow \phi \vee (\psi \wedge \mathbf{Y}(\psi \mathbf{S} \phi)) .$$

## Example (Translating LTL to Propositional Logic $\chi(\cdot)$ )

$$\chi(p \mathbf{U} q) = q \vee (p \wedge \mathbf{x}_{p \mathbf{U} q}) .$$

## Translating LTL to $\omega$ -automata (2)

NUXMV's tableau construction:  $T_\varphi \doteq \langle V_\varphi, \Theta_\varphi, \rho_\varphi, \mathcal{J}_\varphi \rangle$ , where

- Set of Boolean variables:  $V_\varphi \doteq \text{el}(\varphi)$ ,
- Initial condition:

$$\Theta_\varphi \doteq \chi(\varphi) \wedge \bigwedge_{Y_\psi \in \text{el}(\varphi)} \neg Y_\psi,$$

- Transition relation:

$$\rho_\varphi \doteq \bigwedge_{X_\psi \in \text{el}(\varphi)} (X_\psi \leftrightarrow \chi'(\psi)) \wedge \bigwedge_{Y_\psi \in \text{el}(\varphi)} (\chi(\psi) \leftrightarrow Y'_\psi),$$

- Justice set (a fairness condition):

$$\mathcal{J}_\varphi \doteq \{ \chi(\psi \text{ U } \phi) \rightarrow \chi(\phi) \mid X_{\psi \text{ U } \phi} \in \text{el}(\varphi) \} .$$

- Fair states:

$$\mathcal{F}_\varphi^K \doteq \{ s \mid T_\varphi, s \models E \bigwedge_{\psi \in \mathcal{J}_\varphi} GF\psi \} .$$

# The Symbolic Monitoring Algorithm

```

1 function symbolic_monitor( $K \doteq \langle V_K, \Theta_K, \rho_K, \mathcal{J}_K \rangle$ ,  $\varphi(AP)$ ,  $u \in (\Psi(O) \times \mathbb{B})^*$ )
2    $T_\varphi \doteq \langle V_\varphi, \Theta_\varphi, \rho_\varphi, \mathcal{J}_\varphi \rangle \leftarrow \text{ltl\_translation}(\varphi)$ ;
3    $T_{\neg\varphi} \doteq \langle V_\varphi, \Theta_{\neg\varphi}, \rho_\varphi, \mathcal{J}_\varphi \rangle \leftarrow \text{ltl\_translation}(\neg\varphi)$ ;
4    $\mathcal{F}_\varphi^K \leftarrow \text{fair\_states}(K \otimes T_\varphi)$ ;
5    $\mathcal{F}_{\neg\varphi}^K \leftarrow \text{fair\_states}(K \otimes T_{\neg\varphi})$ ;
6    $r_\varphi \leftarrow \Theta_K \wedge \Theta_\varphi \wedge \mathcal{F}_\varphi^K$ ; /* no observation */
7    $r_{\neg\varphi} \leftarrow \Theta_K \wedge \Theta_{\neg\varphi} \wedge \mathcal{F}_{\neg\varphi}^K$ ;
8   if  $|u| > 0$  then /* first observation */
9      $r_\varphi \leftarrow r_\varphi \wedge \text{OBS}(u_0)$ ;
10     $r_{\neg\varphi} \leftarrow r_{\neg\varphi} \wedge \text{OBS}(u_0)$ ;
11   for  $1 \leq i < |u|$  do /* more observations */
12     if  $\text{RES}(u_i) = \perp$  then /* no reset */
13        $r_\varphi \leftarrow \text{fwd}(r_\varphi, \rho_K \wedge \rho_\varphi)(V_K \cup V_\varphi) \wedge \text{OBS}(u_i) \wedge \mathcal{F}_\varphi^K$ ;
14        $r_{\neg\varphi} \leftarrow \text{fwd}(r_{\neg\varphi}, \rho_K \wedge \rho_\varphi)(V_K \cup V_\varphi) \wedge \text{OBS}(u_i) \wedge \mathcal{F}_{\neg\varphi}^K$ ;
15     else /* with reset */
16        $r \leftarrow r_\varphi \vee r_{\neg\varphi}$ ;
17        $r_\varphi \leftarrow \text{fwd}(r, \rho_K \wedge \rho_\varphi)(V_K \cup V_\varphi) \wedge \chi(\varphi) \wedge \text{OBS}(u_i) \wedge \mathcal{F}_\varphi^K$ ;
18        $r_{\neg\varphi} \leftarrow \text{fwd}(r, \rho_K \wedge \rho_\varphi)(V_K \cup V_\varphi) \wedge \chi(\neg\varphi) \wedge \text{OBS}(u_i) \wedge \mathcal{F}_{\neg\varphi}^K$ ;
19   if  $r_\varphi = r_{\neg\varphi} = \perp$  then return  $\times$ ;
20   else if  $r_\varphi = \perp$  then return  $\perp^a$ ;
21   else if  $r_{\neg\varphi} = \perp$  then return  $\top^a$ ;
22   else return ?;
```



# The Symbolic Algorithm: A Sample Run

## Monitoring $p \text{ U } q$ assuming $p \neq q$

$$\begin{aligned} \varphi &\doteq p \text{ U } q \equiv q \vee (p \wedge X(p \text{ U } q)), \\ O &= \{p, q\}, & V_\varphi &= \{p, q, x \doteq X_{p \text{ U } q}\}, \\ \Theta_\varphi &= q \vee (p \wedge x), & \Theta_{\neg\varphi} &= \neg(q \vee (p \wedge x)), \\ \rho_\varphi &= x \leftrightarrow (q' \vee (p' \wedge x')), & \mathcal{J}_\varphi &= \mathcal{J}_{\neg\varphi} = \top, \\ K &= \langle O, p \neq q, p' \neq q', \emptyset \rangle, & u &= \{p\}\{p\} \cdots \{q\}\{q\} \cdots . \end{aligned}$$

## Execution

- 1 Initially (L6–7):  $r_\varphi \leftarrow \Theta_\varphi$ ,  $r_{\neg\varphi} \leftarrow \Theta_{\neg\varphi}$ ;
- 2 Taking  $u_0 = \{p\}$  (L9–10):  $r_\varphi = \Theta_\varphi \wedge (p \wedge \neg q) \equiv p \wedge \neg q \wedge x$ ,  $r_{\neg\varphi} = \Theta_{\neg\varphi} \wedge (p \wedge \neg q) \equiv p \wedge \neg q \wedge \neg x$ . (output is ?)
- 3 On next  $\{p\}$ ,  $r_\varphi$  and  $r_{\neg\varphi}$  remain unchanged (L13–14), as  $\rho_\varphi \wedge (p' \wedge \neg q') \equiv x \leftrightarrow x'$ .
- 4 On next  $\{q\}$ ,  $\rho_\varphi \wedge (\neg p' \wedge q') \equiv x \leftrightarrow \top$ , and  $\text{fwd}(r_{\neg\varphi}, \rho_\varphi)(V_\varphi) \wedge (\neg p' \wedge q')$  (L14) is unsatisfiable, i.e.  $r_{\neg\varphi} = \perp$ . ( $r_\varphi$  is still not empty, output is  $\top^a$ .)
- 5 Taking more  $\{q\}$  does not change the output, unless the assumption  $p \neq q$  is broken:  $r_\varphi = r_{\neg\varphi} = \perp$ , output is  $\times$ .

# Correctness Proof (Sketch)

The function `symbolic_monitor` implements the monitor function  $\mathcal{M}_\varphi^K(\cdot)$ .

Proof.

① Some abbreviations:

$$u \lesssim w \quad \Leftrightarrow \quad \forall i. i < |u| \Rightarrow w_i(V_k \cup AP) \models \text{OBS}(u_i)(O), \quad (1)$$

$$\mathcal{L}_\varphi^K(u) \quad \doteq \quad \{w \in \mathcal{L}(K) \mid (w, \text{MRR}(u)) \models \varphi \wedge u \lesssim w\}, \quad (2)$$

$$L_\varphi^K(u) \quad \doteq \quad \{v \mid \exists w. v \cdot w \in \mathcal{L}_\varphi^K(u) \wedge |v| = |u|\} . \quad (3)$$

② Reduced goal:  $L_\varphi^K(u) = \emptyset \Rightarrow r_\varphi(u) = \emptyset$  and  $L_{\neg\varphi}^K(u) = \emptyset \Rightarrow r_{\neg\varphi}(u) = \emptyset$ .

③ Loop invariants (by induction on the length of  $u$ ):

$$r_\varphi(u) = \left\{ s \mid \exists w \in \mathcal{L}(K \otimes T^\varphi). (w, \text{MRR}(u)) \models \varphi \wedge u \lesssim w \wedge w_{|u|} = s \right\}, \quad (4)$$

$$r_{\neg\varphi}(u) = \left\{ s \mid \exists w \in \mathcal{L}(K \otimes T^{\neg\varphi}). (w, \text{MRR}(u)) \models \neg\varphi \wedge u \lesssim w \wedge w_{|u|} = s \right\} .$$

④ Correctness of reset (Line 13):

$$r_\varphi(u) \vee r_{\neg\varphi}(u) = \left\{ s \mid \exists w \in \mathcal{L}(K \otimes T_0^\varphi). u \lesssim w \wedge w_{|u|} = s \right\} \quad \text{where} \quad T_0^\varphi = \left\langle V_\varphi, \bigwedge_{Y_p \in \text{el}(\varphi)} \neg Y_p, \rho_\varphi, \mathcal{J}_\varphi \right\rangle . \quad (5)$$

## Modifying/Extending the Algorithm

### From Offline to Online Monitors

- 1 Preparing initial belief states  $r_\varphi$  and  $r_{\neg\varphi}$ ;
- 2 LOOP start: taking just one input state  $s$ ;
- 3 Update belief states, getting new  $r_\varphi$  and  $r_{\neg\varphi}$ ;
- 4 Output a verdict in  $\mathbb{B}_4$ .

### From Symbolic to Explicit-State Monitors

- 1 Utilizing the *canonicity* of BDDs: each Boolean function (up to  $=$ ) has an unique address in the memory;
- 2 Constructing monitor automata for all possible inputs;
- 3 Each location of the automaton represent a pair of BDDs:  $(r_\varphi, r_{\neg\varphi})$ ;
- 4 Always terminates due to the finiteness of BDDs.

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## Experimental evaluation - The tool

The monitoring algorithm has been implemented in NuRV - an `nuXmv` extension for Runtime Verification<sup>3</sup>.

### NuRV's features

- “Traditional” RV plus ABRV
- Offline vs. Online
- BDD-based v.s Code Generation
- Reactive vs. Deductive
- Code generation in various programming language;
- Code generation in SMV, allowing formal verifications of the correctness or other properties of the monitor.

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<sup>3</sup>A. Cimatti, C. Tian, and S. Tonetta. [NuRV: a nuXmv Extension for Runtime Verification](#).

In B. Finkbeiner and L. Mariani, editors, *LNCS 11757 - Runtime Verification (RV 2019)*. Springer International Publishing, Porto, Portugal, Oct. 2019

## Model checking SMV monitors

Used `NUXMV` to encode the following properties:

- The *rough* correctness (w/o resets):  
 $(F M._{true}) \rightarrow \varphi$  or  $(F M._{false}) \rightarrow \neg\varphi$ ;
- The monotonicity of monitors:  
 $G M._{unknown} \vee (M._{unknown} U M._{concl})$ ;
- Comparison of two monitors (M1: with BI, M2: w/o BI):  
 $\neg F BI \vee$ , where  $BI \vee := (M1._{concl} \wedge \neg M2._{concl})$ .
- The correctness of resets:  
 $X^n (M._{reset} \wedge X (\neg M._{reset} U M._{true})) \rightarrow X^n \varphi$ .

### Observation

The full correctness of LTL monitors cannot be model checked by LTL itself. (Epistemic operator needed)

## Tests on LTL patterns

We tested on Dwyer's 55 LTL patterns<sup>4</sup>:

- Collected from over 500 specifications from at least 35 different sources.
- 11 groups: Absence, Existence, Bounded Existence, Universality, Precedence, Response (2-causes-1, 1-cause-2), Precedence Chain (2-stimulus-1, 1-stimulus-2), Response Chain, Constrained Chain;
- 5 scopes: Globally, Before, After, Between, After-Until.

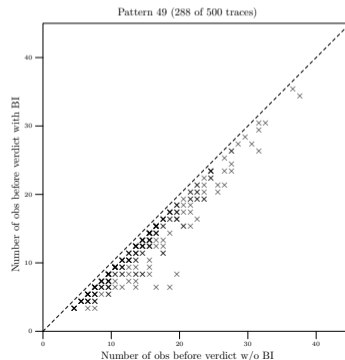
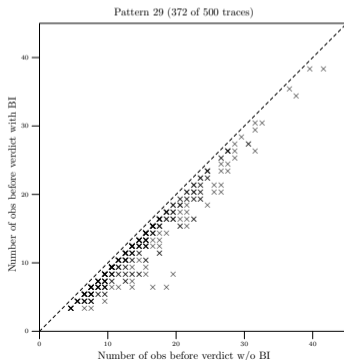
All these patterns can be successfully synthesized into working monitors (with or w/o BI); 500 random traces (each with 50 states) were used to compare the monitoring results.

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<sup>4</sup><https://matthewbdwyer.github.io/psp/patterns/ltl.html>

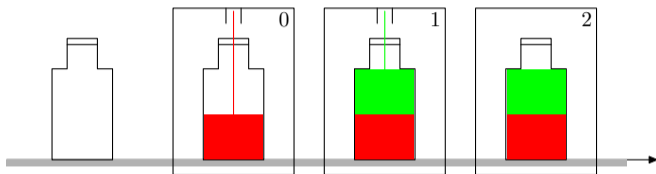
# Tests - The Value of Assumption

- Assumption: transitions to  $s$ -state occur at most 2 times:  $((\neg s) W (s W ((\neg s) W (s W (G \neg s))))))$   
 $(\varphi W \psi \doteq (G \varphi) \vee (\varphi U \psi))$
- Pattern 29:  $s$  responds to  $p$  after  $q$  until  $r$ :  $G(q \wedge \neg r \rightarrow ((p \rightarrow (\neg r U (s \wedge \neg r)))) W r)$ .
- Pattern 49:  $s, t$  responds to  $p$  after  $q$  until  $r$ :  
 $G(q \rightarrow (p \rightarrow (\neg r U (s \wedge \neg r \wedge X(\neg r U t)))) U (r \vee G(p \rightarrow (s \wedge X F t))))$





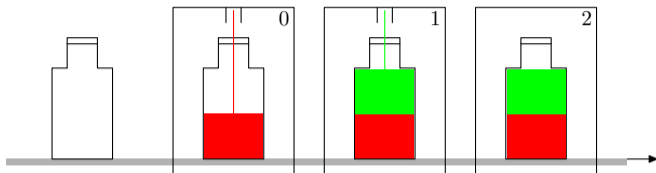
## Tests on a Factory Model (1)



### Model variables:

- `bottle_present[0-2]`: there exists a bottle at position 0-2;
- `bottle_ingr1[0-2]`: red ingredient in the bottle at position 0-2;
- `bottle_ingr2[0-2]`: green ingredient in the bottle at position 0-2;
- `move_belt`: the belt is moving;
- `new_bottle`: new bottle at position 0 before the belt starts to move.

## Tests on a Factory Model (2)



- Whenever the belt is not moving and there is a bottle at position 2, both ingredients are filled in that bottle:

$$\varphi = G ((\text{bottle\_present}[2] \wedge \neg \text{move\_belt}) \rightarrow (\text{bottle\_ingr1}[2] \wedge \text{bottle\_ingr2}[2]))$$

- Two monitors: M1 (with BI), M2 (w/o BI).
- Model checking spec:  $\neg F \text{BI}\nu$ , where  $\text{BI}\nu := (\text{M1}.\_ \text{concl} \wedge \neg \text{M2}.\_ \text{concl})$ .
- Conclusion*: the monitor M1 is predictive, it outputs  $\perp^a$  soon after the fault at position 0.

## Conclusions

- We propose ABRV – an extended RV framework where assumptions, partial observability and resets are supported;
- A new four-valued LTL semantics called ABRV-LTL extended from  $LTL_3$ ;
- A symbolic PLTL monitoring algorithm for ABRV;
- Under certain assumptions: 1) the resulting monitors are predictive; 2) some non-monitorable properties become monitorable.

### Future directions

- ABRV with more expressive (temporal) logics: LTL + metric/epistemic operator(s); FOL (with quantifiers over time); MSOL (WS1S, WS2S);
- MC/SAT-based monitoring algorithms (when BDD is not applicable).

### Question (independent of PLTL)

How to support Assumption/Partial-Observability/Resets in your RV tool/approach?

## Backup: Fair Kripke Structure (FKS)

Let  $\mathbb{B} = \{\top, \perp\}$  denote the type of Boolean values, a set of *Boolean formulae*  $\Psi(V)$  over a set of propositional variables  $V = \{v_1, \dots, v_n\}$ , is the set of all *well-formed formulae* (wff) built from variables in  $V$ , propositional logical operators like  $\neg$  and  $\wedge$ , and parenthesis.

### Definition

Let  $V$  be a set of Boolean variables, and  $V' \doteq \{v' \mid v \in V\}$  be the set of *next state* variables (thus  $V \cap V' = \emptyset$ ). An FKS  $K = \langle V, \Theta, \rho, \mathcal{J} \rangle$  is given by

- $V$ , the set of Boolean variables,
- a set of initial states  $\Theta(V) \in \Psi(V)$ ;
- a transition relation  $\rho(V, V') \in \Psi(V \cup V')$ ,
- a set of Boolean formulae  $\mathcal{J} = \{J_1(V), \dots, J_k(V)\} \subseteq \Psi(V)$  called *justice requirements*.

The *forward image* of a set of states  $\psi(V)$  on  $\rho(V, V')$  is a Boolean formula  $\text{fwd}(\psi, \rho)(V) \doteq (\exists V'. \rho(V, V') \wedge \psi(V))[V/V']$ , where  $[V/V']$  substitutes all (free) variables from  $V'$  to  $V$ .